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A mathematical method for the turbulent behavior of crowds using agent particles

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Abstract Among the people moving as a group there appear social and psychological forces together with physical forces such as friction and resistance. With the definition that the *field of the crowd* is the region of those forces continuously extending with varying strength, and with the pre-requisite that the spatial distribution of the crowd, *i.e.*, the distribution of the field, varies according to the hydrodynamic rule by the Navier-Stokes equation, a methodology was proposed to describe the behavior of the crowd composed of many agent particles as the movement of a compressible, turbulent fluid. A numerical calculation was exemplified for the dynamic behavior and spatial distribution of crowds during movements when there appears a conflict between groups with different characters, imaging for instance the medieval battle of Breitenfeld.

1. Introduction

In our everyday life, we often find the movement of the crowd as if it strongly resembles in the motion of a fluid especially when many people come together to execute a common task [1]. Under some circumstances their movement may be restricted by some social and psychological factors such as norms, bonds, and items under tacit understandings within their own group. Individuals in the group thus make invisible mutual-interaction among them. The collectivity of the crowd becomes high in the case of a strong interaction which restricts the unlimited movement of the individuals. When the population density of the crowd exceeds a certain critical value, a sort of viscous character originating from physical and psychological frictions may appear. Moreover in such a dense crowd the exchange of information is limited only within a local region so that the movement of the crowd in a group may become incoherent. The movement of the crowd, therefore, may be similar to the turbulent motion of a compressible viscous fluid.

When the crowd disperses and the individuality of the people increases, various characters of individuals become to appear discretely so that we must consider the crowd as an agglomeration of independent individuals. When we describe the collective motion of an object consisted of individual particles, one of the useful methods is to adopt the agent particles (or simply, the agents hereafter) [2], and by doing so we have obtained a great deal of valuable knowledge about the behavior of pedestrians, the moving patterns of social insects, the attitude change or the opinion dynamics of individuals, etc [3-8]. In this paper we will also treat the crowd as an agglomeration of agents. The behavior of the crowd, however, will be simulated more realistically if we introduce a unified method by which we possibly represent the motion of the crowd from its dense state to a dilute state throughout the whole process of its behavior while the discrete nature of individual agents is held. Along this line we like to propose a mathematical method to realize the possibility as such by using the Navier-Stokes equation. For the feasibility study of this method, we then try to model the mass

movement of troops with different characters as the fluid motion of the crowd by investigating their behavior during a conflict.

In the next section mathematical method are given and in section 3 several type of problems are considered to which we can apply this method. In section 4 the treatment of forces is described when we simulate the mass motion of the crowd during a conflict by our method, followed by an example calculation in section 5, supposing a medieval battle. Section 6 gives concluding remarks.

2. Turbulent equation for discrete particles and its numerical treatment

Individuals in the crowd interact with tacit, psychological forces which mutually impose social restrictions on each other, even if they do not physically contact, through the inspection of surrounding situations and their colleague's movement. Such situation can be reworded as that the spatial extension of the crowd just corresponds to the field where such fluid forces extend. When the individual in the crowd makes a relative motion to each other, it may give rise to the viscosity and the turbulence in the group, resulting in the distortion of the force field. Although the local strength of the field changes according to the change of the distribution of the crowd, the field continuously extends from the inner region of the crowd to the outward. We have been accustomed to the consideration that the field made by interacting particles is a sort of fluid, such as the fluid model of nucleus, the droplet model of nuclear fission, the field of quarks and gluons and so on. The fact that the granular materials containing inter grain water act as a fluid to a certain limit [9-11] is quite indicative from our viewpoint. If the flow of the crowd really resembles in the flow of the field, individuals in the crowd can be said to move according to the kinetic rule determined by the field. When we replace the crowd with representative agents, we can deduce the strength of the local field in terms of the local density of the agents whose flow is determined by the kinetic rule of the field. According to such a conception, introducing the agents as many number as they are proportional to the number of people in the real crowd, we estimate the movement of the crowd as the flow of the filed at the position of each agent by using the turbulent equation.

The behavior of a force field, *i.e.* the behavior of people or the agents in our case, is given by the following turbulent equation of compressible fluid and the equation of continuity.

$$\frac{\partial \mathbf{V}}{\partial t} = -(\mathbf{V} \cdot grad)\,\mathbf{V} + \frac{\eta}{\rho}\Delta\mathbf{V} + \frac{1}{\rho}(\varsigma + \frac{\eta}{3})grad \cdot div\,\mathbf{V} - \frac{1}{\rho}grad|\mathbf{P}| + \frac{\mathbf{F}}{\rho} \qquad (1)$$

$$\frac{\partial \rho}{\partial t} = -div(\rho\mathbf{V}) \qquad (2)$$

where **V** is the velocity of the crowd, ρ the density, η the coefficient of viscosity, ζ the volumetric viscosity, and **P** and **F** are respectively the pressure and the force acting on the crowd.

When we intend to simulate the collective motion of the crowd, one of the inherencies to be considered is the finiteness of moving velocity. Hence, although we obtain the fluid velocity of the agent from equation (1), it should be limited to a certain maximum value V_{max}. Moreover the viscosity arising from the crowd motion intuitionally seems to increase with the crowd density so that we set simply as $\eta/\rho \equiv H$ and $(\zeta + \eta/3)/\rho \equiv Z$, and treat the quantities H and Z as constants as a first approximation. Moreover since the fourth and fifth terms on the right hand side of equation (1) give forces acting on the unit mass of the crowd, those terms give values proportional to the force acting on an individual agent when we initially distribute the agents with the number just in proportion to the local density of the crowd. We therefore omit the factor ρ in the fourth and fifth terms of equation (1). Furthermore equation (2) is no more required so far as the number of agents is conserved.

The two-dimensional velocity of the agent m on the direction k (=x and y), $V_{m,k}$, is now given by

$$\frac{\partial V_{m,k}}{\partial t} = -V_{m,k} \cdot \frac{\partial V_{m,k}}{\partial k} - V_{m,\ell} \cdot \frac{\partial V_{m,k}}{\partial \ell} + H \cdot \sum_{q=k,\ell} \frac{\partial^2 V_{m,q}}{\partial q^2} + Z \cdot \frac{\partial}{\partial k} \sum_{q=k,\ell} \frac{\partial V_{m,q}}{\partial q} - \frac{\partial P_m}{\partial k} + F_{m,k} \qquad (3)$$

where ℓ=y and x, and P_m and $F_{m,k}$ are respectively the pressure and the force acting on the agent m. When every agent is a member of one of the groups, we deduce the partial derivative of $V_{m,k}$ with

respect to the coordinates k as

$$\frac{\partial V_{m,k}}{\partial k} \cong (\langle V_{m,k} \rangle_+ - \langle V_{m,k} \rangle_-)/(d_{k+} + d_{k-}) \qquad (4)$$

Here $<V_{m,k}>_+$ and $<V_{m,k}>_-$ are the averages of the weighted sum of the agents belonging to the same group within the respective sides of +k and –k of the circle of a radius R_O as given by

$$\langle V_{m,k} \rangle_{\pm} = \pm \sum_j {}' V_{j,k} \exp(\theta_{mj}\mathbf{i}) \cdot \exp(-r_{mj}/\lambda) \qquad (5)$$

where Σ' represents the sum without including the m itself, $\theta_{mj} = \arg(r_{mj})$, $\mathbf{i}$ is the imaginary unit vector. Moreover the quantity $d_{k\pm}$ is the average of the distance on the $\pm k$ directions for all agents within the half circles on the +k and –k respective sides as given by

$$d_{k\pm} = \frac{1}{n_{k\pm}} \sum_j {}' X_j \exp(\theta_{mj}\mathbf{i}) \qquad (6)$$

where $n_{k\pm}$ are the total number of agents within the respective half circles. The weight in equation (5) is assumed to exponentially decrease with the radial distance, and λ is the attenuation distance.   The regions $\Sigma_{x\pm}, \Sigma_{y\pm}$ where the Σ' on the sides of $\pm x$ and $\pm y$ should be taken are depicted in Fig.1.

On the other hand the second partial derivative of $V_{m,k}$ is given by

$$\frac{\partial^2 V_{m,k}}{\partial k^2} \cong \left(\left\langle \frac{\partial V_{m,k}}{\partial k} \right\rangle_+ - \left\langle \frac{\partial V_{m,k}}{\partial k} \right\rangle_- \right)/(d_{k+} + d_{k-}) \qquad (7)$$

Here $< \partial V_{m,k}/\partial k >_{\pm}$ can be derived by replacing $V_{j,k}$ in equation (5) by $\partial V_{j,k}/\partial k$. The other second derivatives are also given by the same manner. The values of partial derivatives of V, $\partial V_{m,\lambda}/\partial k$, $\partial^2 V_{m,\lambda}/\partial k^2$ and $\partial^2 V_{m,\lambda}/\partial k \partial \ell$, asymptotically approach null when the number density of agents decreases so that equation (3) approaches the usual equation of motion for a point mass. Discretizing equation (3) with respect to time, we have the velocity component $V^t_{m,k}$ of the agent m in the direction k at a time t as

$$V^t_{m,k} = \min\left(V^{t-\Delta t}_{m,k} + \left(\frac{\partial V_{m,k}}{\partial t} \right)^{t-\Delta t} \Delta t, \; V_{\max} V^{t-\Delta t}_{m,k} / \left| V^{t-\Delta t}_{m,k} \right| \right) \qquad (8)$$

Here the superscript t indicates the value at a discrete time t, and the quantity Δt is a time-step. Using this quantity, we derive the position of m in the direction k at t, $X^t_{m,k}$, as

$$X^t_{m,k} = X^{t-\Delta t}_{m,k} + 0.5(V^t_{m,k} + V^{t-\Delta t}_{m,k})\Delta t + 0.25(\frac{\partial V^t_{m,k}}{\partial t} + \frac{\partial V^{t-\Delta t}_{m,k}}{\partial t})(\Delta t)^2 \qquad (9)$$

3. What type of problems can we simulate with this method ?
In the case of the movement of groups with different characters, conflicts will possibly occur if the space where one group invades is already possessed by the other group. In such a case the extinction of one of the groups, the homogenization of a group to the other, or the expulsion or the flight of the group to the outer region will occur as a result of the conflict. Some studies on such subjects have been done about the rise and fall of a community by using an agent model [16, 17], and the battle between medieval armies at Agincourt using a continuity theory [18].

With regard to the application of our model to the competing social phenomena, we can think of the following cases which have a common intention of motion of the individuals each of which movement is, however, seemingly random, such as (1) the behavior of the groups of wild animals which cause rivalries over their territory, (2) the behavioral pattern of people in the street demonstration and the adversarial people regulating their movement, (3) the behavior of soldiers in the troops during the battles in a close combat, especially before the medieval era, and so on. Since there
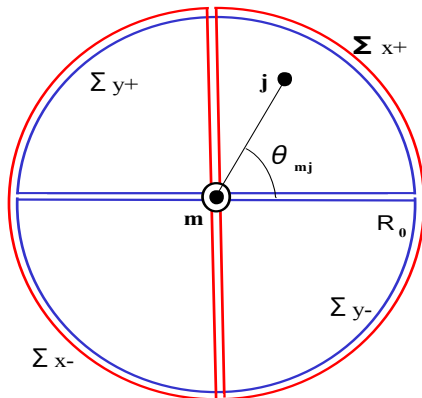
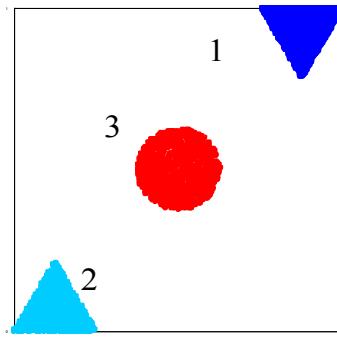**Figure 1.** Regions to be performed for summation

Figure 2. Initial distribution of agents.

exist quite few documents for those real competitions on the movement of individual people or animal, which describe the gradual shift of motion from a laminar flow with a high density of the crowd to a turbulent flow of a low density, direct comparison of the simulation with real situations is difficult to make. Notwithstanding the rareness of described documents, we imagine the situation of the battle of Breitenfeld in Germany, for instance, as follows [12, 13, 14].

   In 1631, two armies were confronting each other at Breitenfeld, the one was the joint army of Sweden and Saxony headed by King Gustav, whose center were constituted from many battalions with cavalries and maskets on both wings. The other was the Imperial army led by Tilly whose center was formed from a number of Tercios also with cavalries and maskets on both wings. The Tercio is the combat unit composed of infantries with spears in its center with maskets on all sides around it.

   The movement of those infantries was of a quite streamline flow at the beginning as well as the movement of cavalries. The center of Imperial battalion first forwarded to attack the enemy. Although its left wing was intercepted by the Swedish cavalries and maskets to result in running back in a scattered manner, the right wing of Imperial army made the Saxony army run backward. The Swedish Gustav made his army on the right wing move to the left, moreover made the cavalries which were standing by on the right wing carry the attack all the way to the weak central part of Imperial army finally to succeed in breaking it into pieces. Once it occurred the close battle between infantry and/or cavalries, the battle field became to be of turbulence and infantry fell into serious disorders to flee randomly, whose situation can also be imagined from the print of those days [15]

   In the next section assumed forms are described regarding the forces acting between the agents to simulate such conflicts by our method.

4. Example formulation of forces

The pressure **_P_** in equation (1) is for the external pressure originating from the direct and indirect coercion and the psychological fear due to the enemy in the case of conflict. Such a pressure may be given as a function of the distance $|X_g\text{-}X|$ between the adversarial group and the agent itself as

$$\mathbf{P} = \xi_0 \sum_{\phi} (\mathbf{X} - \mathbf{X}_{\phi g}) \Big/ \left| (\mathbf{X} - \mathbf{X}_{\phi g}) \right|^{\alpha} \tag{11}$$

where ξ_0 and α are constants, **_X_** is the position vector of the agent, $\mathbf{X}_{\phi g}$ is the position corresponding to the gravitational center of the adversarial group ϕ. We simply take $\alpha=3$ in the example calculation.

   On the other hand the force **_F_** is consisted from:(1) the resistance **_f₁_** against the isolated existence away from the agent's group, (2) the cohesive force **_f₂_** originating from the mutual dependence and the solidarity within the same group, and from social norms restraining agents, (3) the repulsive force **_f₃_** originating from the population pressure and the exclusion due to the conflict within the same group,

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(4) the belligerent force f_4 against the distant enemy when each agent moves toward the gravitational center of adversarial group far from it, (5) the belligerent force f_5 against the nearest member of the enemy which approaches the agent within a critical range r_c, (6) the force f_6 to run away from the gravitational center of the enemy when the enemy is far from the group to which the agent belongs, and (7) the force f_7 to run away from the nearest member of the enemy when it approaches the agent within a range r_c.

The force f_1 is assumed, as the pressure P, to be proportional to some powers of distance $|X_g-X|$ as $f_1 = \xi_1(X_g-X)|X_g-X|^\beta$, where ξ_1 and β are constants, and X_g is the vector for the gravitational center of the group, setting $\beta=1$, for instance, in what follows.

The forms of $f_2 \sim f_7$ are all simply assumed to have an exponential dependence on the distance.   At the position X of the agent m the forces f_2 and f_4, for instance, are respectively given by

$$\mathbf{f}_2 = \xi_2 \cdot \gamma_m \sum_\kappa \gamma_\kappa \varepsilon_{m\kappa} \exp(-\Psi_2 r_{m\kappa}) \tag{12}$$

$$\mathbf{f}_4 = \varepsilon_{m\Phi} \xi_4 \exp(-\Psi_{4\Phi} r_{m\Phi}) \tag{13}$$

Here γ_m is a coefficient, as will be described later, which relates to the strength of the force for the agent m as a group leader, ξ and Ψ with suffixes are all constants, $r_{m\kappa}$ is the distance between the agents m and κ, $\varepsilon = r/r$, the suffix κ indicates the other agents within the same group, ϕ represents the nearest agent in the other groups which approaches m, and Φ represents the gravitational center of the other group. To cope with the infinite approach of people within the crowd, a strong repulsive force is assumed between two agents when the distance r between them becomes less than r_a.

Generally speaking there exists more than one leader in a group whose members usually follow them. The leaders draw closer the group members as attractors, but they apt to become independent from the other leaders or antagonistic to each other, intending to acquire the power and supremacy. To model such a situation, we introduce 5 leaders (or attractor agents) in each group and invest in them the cohesive force f_2 with γ_m (1) times stronger than the force acting between the ordinary agents. Moreover the force is assumed to become repulsive between the leaders; γ_m - γ_m.

When the agents belonging to different groups closely approach each other, a conflict caused by the forces such as (12) and (13) may probably arise. In the case of the approach within the distance less than r_c, the agents belonging to the weaker group is assumed (i) to become extinct with a probability p_1, (ii) to be homogenized or to adapt themselves to the stronger group with p_2, and (iii) to safely exist as before with p_3, where $p_1+p_2+p_3=1$. The resultant effects of the extinction and the change in characters of the agents are fed back to the movement of the system as a fluid.

5. Simulation of a conflict

The agents are assumed to move within a rectangular region of a side length $L(=1.0)$, and to randomly distribute at an initial time in some limited regions 1, 2 and 3 where the different groups of agents 1, 2 and 3 with the number of these groups $n_1=2000$, $n_2 = n_3 =1000$ are respectively exist, as shown in figure 2..We also assume the offensiveness increases and the flight character decreases both in the order of the group 3, the group 2 and the group 1. We use here the following values of constants and parameters as a base case: $R_0=0.0625$, $\lambda=0.0313$, $r_c=0.02$, $r_a=0.001$, $V_{max}=0.141$, $\gamma_m=900$, $H_1=H_2=2\times10^{-4}$, $H_3=5\times10^{-5}$, $Z_1=Z_2=2.5\times10^{-4}$, and $Z_3=5.2\times10^{-5}$. The Reynolds number is $Re = V_{max}L/\eta \simeq V_{max}L/H_1 \simeq 10^3$, which corresponds to a developed turbulence.

The simulation is shown in figure 3. The values of constants regarding the forces appeared in the previous section are all taken common for these (a)~(d) in figure 3, which seem unessential for the comparison of qualitative results. The second column (b) in this figure shows the base case for a time sequence $t =50, 75, 100, 125$ and 200, where the time means the number of time step. The values of parameters used in this base case give the comparable strengths of viscosity and turbulence. The offensive agents 1 swiftly shift their positions changing their appearance to run down the weak, and the running-away agents 2 and 3 change their outward form to effectively flee from the enemy. The
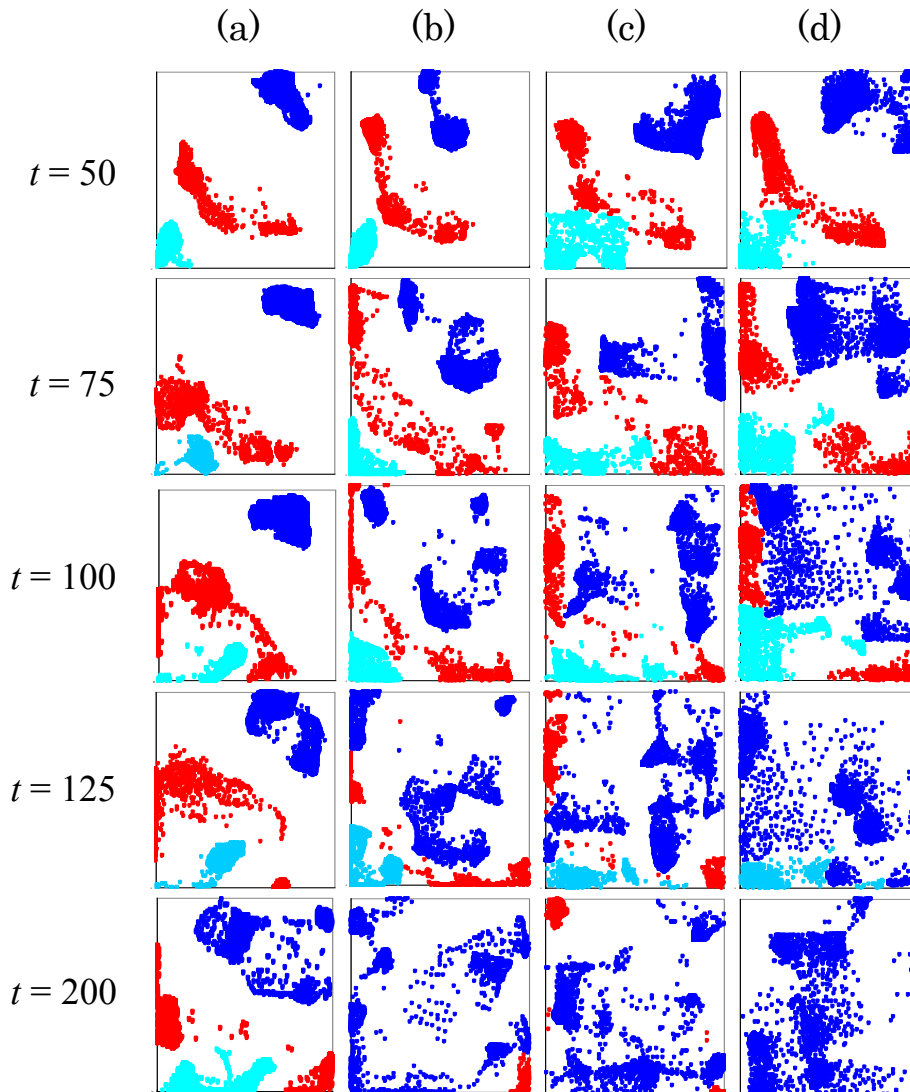
Figure3.:Time behavior of the group 1 (blue), 2 (light blue), and 3 (red) during the battle, where the group 2 and 3 being extinct by the group 1 and the group 3 by the group 2 both with a probability 1.0 when the close encounter takes place between them. Column (b) is the base case, whereas (a) is the case with the viscosity two times higher than (b), (c) is the non-viscous case and (d) is the case obtained with the ordinary equation of motion. The movement of those agents can be seen in animations [19].

offensive agents 1 form a wide wing during $t = 125 \sim 200$ to enclose the weeks, showing the solidarity as a crowd [19].

   The column (c) is for the case $\eta = \zeta = 0$ but $(\mathbf{v} \cdot \text{grad})\mathbf{v} \neq 0$ in the Navier Stokes equation which reduces to the Euler equation. In the case of zero viscosity there does not exist any interaction and hence any social and psychological fields between agents. Although this causes the larger velocity of agents than the base case (b), it appears distinct edges of moving mass with a constant velocity because of the existence of a maximum velocity V_{max}. Furthermore the case (d) corresponds to the ordinary equation of motion with zero viscosity, eliminating the turbulent term, $(\mathbf{v} \cdot \text{grad})\mathbf{v} = 0$. The agents are not subject to any inner forces as physical and psychologically frictional forces, but only the outer forces. The dispersive nature of agents is strong so that their coherency is weak, showing a tendency of individual

movement especially of the agents 1. Since the moving velocity becomes large, the clear edges of moving mass also appear in this case as in the case (c). The case (a), on the other hand, is for the viscosity two times stronger, $H_1=H_2=4\times10^{-4}$, $H_3=1\times10^{-4}$, $Z_1=Z_2=5\times10^{-4}$, and $Z_3=1.05\times10^{-4}$, than the base case (b). In this case the interactive force between agents becomes strong and the agents coagulate, indicating strong collectivity. Furthermore, since the viscosity is strong comparing with the turbulent nature, the moving velocity decreases to result in the motion of the agents as a group.

The viscosity thus acts as a factor to strengthen the coherency of the crowd. Such viscosity depends on the characteristics of the crowd as a condensed matter, namely on the strengths of physical friction and social and psychological interactions. Although there exist some examples of hydrodynamic analyses for the medieval battle of that type [1,18], it is also unclear whether the deformation of the crowd pattern as shown in figure 3 emerges in reality. For the clarification of those points and for making our model a quantitative tool we require the field observation regarding the collective motion of the crowd and close comparison between them and the simulations.

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6. Conclusions

There are so many cases in social sciences where individual elements with different characters gather as a group to act for one common purpose such as the movement of races and refugees, the conflicts and the battles between races and tribes, and some types of demonstration. Our method in this paper can show a solution for such questions as how the crowd exerted by internal and external pressures diffuses in time and space, how its expansion changes in the case of the encounter with the other crowds with different characteristics. Such problems are interesting in that their academic discipline is just at around the midway of social sciences and natural sciences. It is also interesting to make this method more precise by using the behavioral data of the crowd observed in the real field.

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