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A mathematical model for the occurrence of historical events

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A mathematical model for the occurrence of historical events

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Abstract. A mathematical model was proposed for the frequency distribution of historical inter-event time τ. A basic ingredient was constructed by assuming the significance of a newly occurring historical event depending on the magnitude of a preceding event, the decrease of its significance by oblivion during the successive events, and an independent Poisson process for the occurrence of the event. The frequency distribution of τ was derived by integrating the basic ingredient with respect to all social fields and to all stake holders. The function of such a distribution was revealed as the forms of an exponential type, a power law type or an exponential-with-a-tail type depending on the values of constants appearing in the ingredient. The validity of this model was studied by applying it to the two cases of Modern China and Northern Ireland Troubles, where the τ-distribution varies depending on the different countries interacting with China and on the different stage of history of the Troubles, respectively. This indicates that history is consisted from many components with such different types of τ-distribution, which are the similar situation to the cases of other general human activities.

1. Introduction

A historical event in itself does not occur independently. Its occurrence is originated from some cause and the event thus appeared becomes a cause of another successive event. Historical events are therefore continuous in time so that they make the cause and effect to each other. If it really is the case, two events that successively occur must have a certain rule, but what a concrete rule? When the event occurs independently to others, its occurrence obeys the Poisson distribution where the time interval between two events obeys an exponential function. According to the statistics of the wars in the past by Richardson [1], although their occurrence rates approximately obey the Poisson distribution, there exist some periods when the statistics do not seem the case. Moreover the probability for the occurrence of wars seems to have varied from time to time [1]. For the general historical events, however, a statistical treatment as such has not been done so that their distinctive features in statistics are unclear. An approach to such a problem as the statistical treatment of historical events will be made here by introducing the chronological table as a data base.

  In that table chronological events are listed in the order of occurrence. The following matters must be satisfied as it is used as the data base; (1) important facts and their relevant events must be objectively took up without any bias, (2) the extent of the significance of each event is indicated or it can be derived by some means, and (3) the selection of the event must not be so precise nor so coarse for the convenience of their statistical treatment. As for examples, we adopt here the following two chronological tables in modern histories where the events and their importance seem to have been sufficiently and objectively documented: (1) History of Modern China in 1941-2008 [2] and (2) Northern Ireland Troubles in 1968-1999 [3].

   In the following section statistical traits regarding those histories derived from the tables are
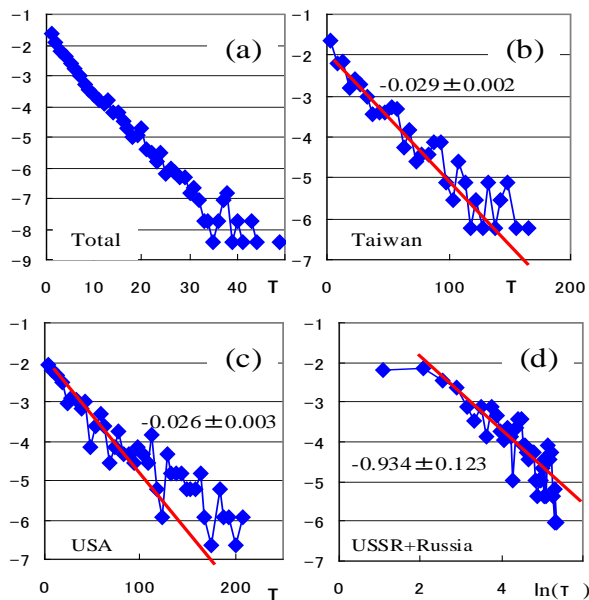
Figure 1. Logarithmic distribution of the inter-event time τ in Modern China (a) as a whole; only for the foreign relations with (b)Taiwan, (c) USA, and (d) USSR and Russia.
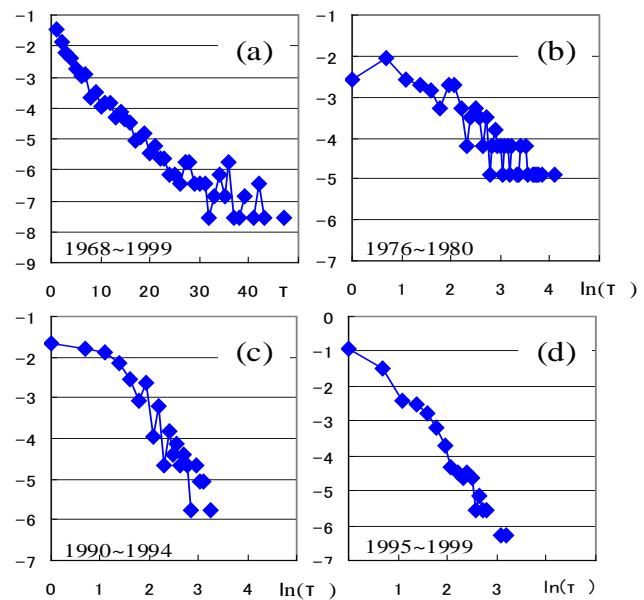
Figure 2. The same as figure 1 but in Northern Ireland Troubles (a) as a whole; only for the period of (b) 1976~1980, (c) 1990~1994, and (d) 1995~1999.

described, and in section 3 a model is proposed for the explanation of those traits. Section 4 is a mathematical description of the model, followed by a conclusion in section 5.

## 2. Chronological tables as historical material

The material [2] comprehensively collects the events occurred in Modern China for all genres of politics, economics, cultures, foreign affairs, military affairs, and social affairs, whereas the material [3] is for the collection of events restricted only to the Troubles. In these tables description is made for each event from one line to some tens of lines to explain the event, which is usable as a measure of the importance of the event, so that we adopt the number of explanation lines as the magnitude of its significance. The collected numbers of events in [2] and [3] are 4542 and 1913, respectively, which correspond to the average inter-event times 5.46 and 5.82 days/event, respectively.

The normalized distributions of the inter-event time τ between two successive events in the materials are respectively shown in a logarithmic scale in figures 1(a) and 2(a), while figure 3 shows the magnitude distribution of the events. In the case of [2], the τ (in days) exponentially distributes in a wide range of τ<40, whereas it is also exponential in 7<τ<30 in the case of [3]. Those events, therefore, seem to have randomly occurred with time at a first glance. The magnitude w (in the number of lines) of the events also seems to exponentially distribute in w<20 if we take the segregation effect into account in the low significance region in w<3~4. When we treat the histories not comprehensively as a whole but partially within each genre of histories or within a limited time period, however, the τ differently distributes from the entirety. Figures 1(b), (c) and (d) show the τ-distributions of the foreign affairs of China with Taiwan, USA, and USSR and Russia, respectively, where we see the subtle difference of the distribution with the different partner. Figures 2(b)~(d) show, on the other hand, the cases during different stages of the Troubles, where we can see the different features of distribution from each other with different time periods.

Further to see the situation of the Troubles, the power of the τ-distribution in each year is shown in figure 4, where a power-law distribution of τ is assumed for every year in $τ_{min}$<τ. Such a power is determined as the value to minimize Kolmogorov-Smirnov statistic together with the value of lower limit $τ_{min}$, by using the method of Goldstein *et al.*[4]. The $τ_{min}$ was determined as 3~6 days for every
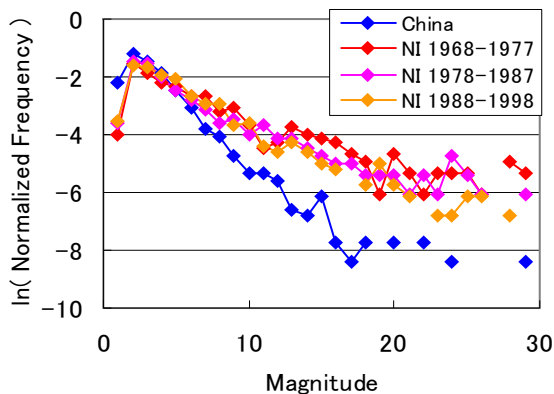
Figure 3. Magnitude distribution of events. NI is the Northern Ireland Troubles for which the distributions are separately shown for three periods.
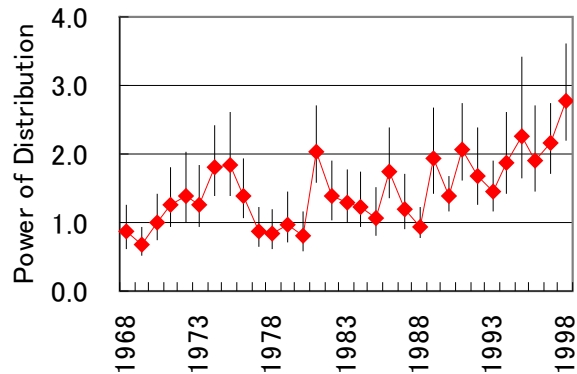
Figure 4. Powers of the distribution of τ in $\tau > \tau_m$ in the Northern Ireland Troubles. Vertical lines indicate the 95% confidence levels.

year. The upper and lower limits shown in this figure represent the 95% confidence levels, which are determined by deriving 2500 model distributions by the Monte Carlo [4]. This figure also indicates the subtle change of the τ-distribution with the progress of the situation.

To investigate what factors are acting on the mechanism that brings successive events into existence, we next introduce a model for the occurrence of historical events.

3. A model for the occurrence of historical events

3.1. Model
When an event occurs in the society, it makes the various stake holders (SHs) relevant to its problem to be in excited states. Here the SH is a general name of constituents who have an interest in the problem and make some type of reactions to thr action regarding to the problem exerted from the outside. When the action the SH receives is of a certain current and common topic for the society, the reaction that the SH makes to the action may become a new action for the other SHs to result in the continuous propagation of such an action-reaction network in the society as schematically shown in figure 5(a). In those actions the event that induces many significant reactions becomes to be cited as a historical event in the chronological table afterward.

Figure 5(b) shows a basic ingredient of such a process, where the magnitude w indicating the significance of the action. In this figure the action originating from the SH_1 stimulates or excites the SH_2 that makes a reaction after a time τ. Here the society is closed so that the information on the action is supposed to propagate within a short time period throughout the society. This seems to be an appropriate prerequisite for the modern society where there exist many types of media that make information spread in the society almost instantaneously as compared with the τ. After the occurrence of a significant event, some events relevant to it occur in a cascade manner, their magnitudes depending only on the magnitude of an event just before the event without depending on the magnitude of the original event itself. Such an assumption seems to be the case at least in the exchange of terrorism [5].

We set E as the discipline or genre such as politics or economics to which the SH_2 belongs, $\varphi(w)dw$ as the probability of an action with the magnitude $[w, w+dw]$ to occur within a unit time, w' as the magnitude of the influence on SH_2, $\Phi_{E,SH2}(w'';w)dw''$ as the probability for the SH_2 stimulated with $[w'', w''+dw'']$ that arises from the action with w, and $\psi(\tau;w')d\tau$ as the probability of a reaction to be made by the SH_2 after a time τ. With the elapse of time, the amazement to an occurred event becomes dim in general and the necessity of a prompt reaction decreases since the significance and the meaning of an initial event change with time. Between the variables w'' and w', therefore, there appears an
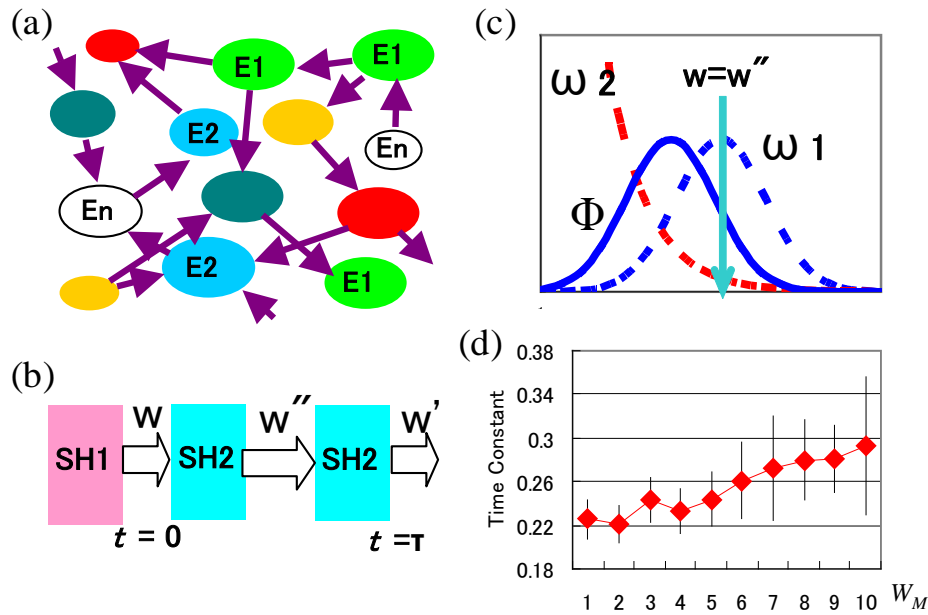
Figure 5. (a) Schematics of action-reaction network. Arrows indicate the direction of the influence which successively comes into existence, (b) basic ingredients of (a), (c) schematics for the relation of Φ, ω_1 and ω_2, (d) time constant $\lambda(w_M)$ for the exponential decrease with τ for the number of event $N(w \geq w_M, \tau)$.

effect of the oblivion of memory and the decrease of impression, so that

$$w' = \eta(\tau) \cdot w'' \tag{1}$$

We call $\eta(\tau)$ as the forgetting function in the followings. The number of events which occur during [τ, τ+dτ], d$N(\tau)$, is given by

$$\frac{dN(\tau)}{d\tau} = \sum_{E} \sum_{SH2} \int_{w_m}^{\infty} \int_{w_m}^{\infty} \psi_{E,SH2}(\tau; w') \, \Phi_{E,SH2}(w''; w) \, \varphi(w) \, dw \, dw' \tag{2}$$

$$\cong K \int_{0}^{\infty} \int_{0}^{\infty} \overline{\psi}(\tau; w') \, \overline{\Phi}(w', \tau; w) \, \overline{\varphi}(w) \, dw \, dw' \tag{3}$$

where K is a constant and w_m is a minimum magnitude of the event that occurs historically ($w_m \cong 0$). The quantities $\overline{\Psi}$, $\overline{\Phi}$ and $\overline{\varphi}$ are the average values throughout the E and SH, all of which are given without the upper bars for simplicity hereafter.

3.2. Mathematical formulation

Here we derive the concrete form of equation (3) by using the data of Modern China and Northern Ireland Troubles and study under what condition of parameters it can reproduce the real τ-distribution.

According to figure 3, the distributions of the magnitude of events that occurred during three different periods of Troubles are substantially the same as each other within a margin of error. Hence an assumption will be accepted such that the distribution of the magnitude of events does not depend on time when they occur during a short time period of history, for instance for several tens of years. In this case, the quantity $\varphi(w)$dw is given by

$$\varphi(w)dw = a \exp(-bw)dw \tag{4}$$

where a and b are parameters. On the other hand the SH_2 that perceives the action of magnitude w is supposed to make a reaction of a similar magnitude to w as it is imagined from the battles between the Catholic and Protestant camps in the Troubles [6] and from the exchange of arms under some ethnic

disputes [7] although its quantitative feature is unclear. Hence we assume the probability $\Phi(w''; w)dw''$ as the product of a normal distribution function $\omega_1(w'';w)dw''$ and a compensation factor $\omega_2(w'')$ to revise it as

$$\Phi(w'';w)dw'' \equiv \omega_1(w'';w) \cdot \omega_2(w'')dw'' \tag{5}$$

$$\omega_1(w'';w) = (c/\pi)^{1/2} \exp\left\{- c(w - w'')^2\right\} \tag{6}$$

$$\omega_2(w'') = d \exp(-ew'') \tag{7}$$

where c, d and e are constants. Such a situation is schematically shown in figure 5(c).

Since the process for the SH_2 to be first stimulated to the magnitude w'' to the time when it give rise to a reaction is restricted only to the inside of the SH_2, not depending on the other SHs, it may be a Poisson process. Hence in this case the function $\psi(\tau;w')d\tau$ is given by

$$\psi(\tau;w')d\tau = f\lambda \exp(-\lambda\tau)d\tau \tag{8}$$

where $\lambda \equiv \lambda(w')$ and f is a constant. When the number of the event with the magnitude $w \geq w_M$, $N(w \geq w_{M;}\tau)$ is plotted as a function of τ in the case of the Troubles, it shows an exponentially decreasing function with τ. Here w_M ia a constant parameter (figure 3 just corresponds to the case of $w_M=1$). From such a figure we can derive the time constant λ with respect to τ for a fixed value of w_M. Then by varying the value of w_M from 1 to 10, we obtain the $\lambda(w_M)$ as a function of w_M as shown in figure 5(d), according to which the $\lambda(w_M)$ is approximately proportional to w_M, and therefore the $\lambda(w')$ is also expected to be proportional to w'. Namely

$$\lambda \equiv \overline{\lambda}_{E,SH2}(w') = g + hw' \tag{9}$$

where g and h are constants. Since the magnitude w'' becomes large when the w' is large, equation (9) means that the larger becomes the magnitude or the shock SH_2 receives by an action, the faster appears the reaction by the SH_2.

Substituting the above functions into equation (2) and performing the integration, we obtain

$$\frac{dN}{d\tau} = \frac{Kadfg}{2} \exp\left(\frac{b^2}{4c}\right) erfc\left(\frac{b}{2c^{1/2}}\right)\frac{\eta\exp(-g\tau)}{b+e+h\eta\tau}\left(G + \frac{bhH\eta}{2cg}\right) \tag{10}$$

where

$$G \equiv 1 - \exp\left(-\frac{b(b+e)}{2c}\right) \cdot \left(erfc\left(\frac{b}{2c^{1/2}}\right)\right)^{-1} \cdot G' \tag{11}$$

$$H \equiv 1 - \frac{b\alpha - 2c^{1/2}}{b\alpha}\exp\left(\frac{b-\alpha}{2c^{1/2}} - \beta\right) - \exp\left(\frac{\alpha^2}{4} - \beta\right) \cdot \left(erfc\left(\frac{b}{2c^{1/2}}\right)\right)^{-1} \cdot H' \tag{12}$$

Here $\alpha \equiv c^{-1/2}(b+e+h\eta\tau)$, $\beta \equiv bc^{-1/2}\alpha/2$, G' and H' are variables (which are given in the Appendix) including constants $a\sim f$, and α and β, and $erfc(x)$ is the conjugate error function. Equation (10) can be rewritten as

$$\frac{dN}{d\tau} \propto \frac{\eta\exp(-g\tau)}{b+e+h\eta\tau}F(\eta,\tau) \tag{13}$$

where $F(\eta,\tau) \equiv G+bhH\eta/(2cg)$.

As seen in equation (13), the distribution function of τ is mainly determined by the constants b,c,e,g and h and the form of the forgetting function η. Although we can not make clear the form of η for long-term memory as a function of time even now, it is known that there does not appear any serious inconsistency when it is assumed as a power-law function of time [8-11]. Hence we also assume as

$$\eta = \gamma\left(\frac{\tau_0}{\tau + \tau_0}\right)^k + (1-\gamma) \tag{14}$$

where γ is the oblivion fraction when $\tau\rightarrow\infty$ (hence $1-\gamma$ is the fraction that remains in the memory), k and τ_0 are constant.
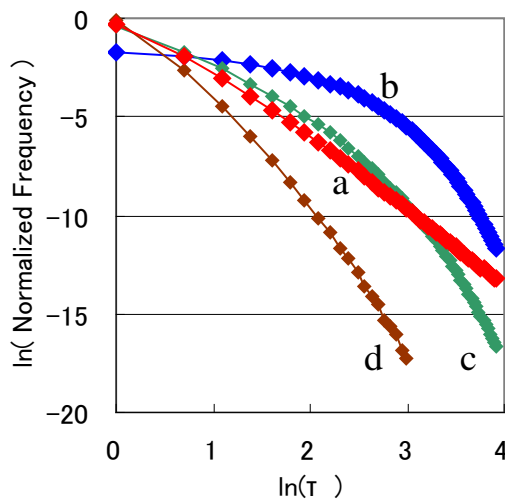
Figure 6. Model calculation for the distribution of τ with $a=28$, $d=0.25$, $c=(d/\pi)1/2$, $f=1.0$ and (a) $b=e=0.2$, $g=h=0.0001$, $\gamma=1.0$, $\tau_0=1.0$, $k=2.0$, (b) $b=e=0.2$, $g=0.2$, $h=0.01$, $\gamma=0.0$, $\tau_0=10.0$, $k=2.0$, (c) $b=e=0.002$, $g=0.2$, $h=0.01$, $\gamma=0.0$, $\tau_0=1.0$, $k=2.0$, and (d) $b=e=0.2$, $g=0.2$, $h=0.0001$, $\gamma=1.0$, $\tau_0=1.0$, $k=3.0$.

When we restrict the values of parameters within each probable range, the factor G becomes a small value whereas the H becomes a factor almost independent on τ. Hence equation (13) approximately becomes

$$\frac{dN}{d\tau} \propto \frac{\eta^2 \exp(-g\tau)}{b+e+h\eta\tau} \tag{15}$$

Depending on the relative magnitude of constants, equation (15) has the following forms of distribution

$$\frac{dN}{d\tau} \propto \exp(-g\tau) \quad \text{when } h \text{ is small and } (\gamma \text{ or } \tau_0^{-1}) \text{ are small} \tag{16}$$

$$\propto \eta\tau^{-1} = \tau^{-k-1} \quad \text{when } g \text{ is small and } \tau \text{ is } l\arg e \tag{17}$$

$$\propto \tau^{-2k} \exp(-g\tau) \quad \text{when } h \text{ is small} \tag{18}$$

$$\propto \tau^{-1} \exp(-g\tau) \quad \text{when } \tau_0 \text{ is } l\arg e \text{ or } \gamma \text{ is small} \tag{19}$$

To ascertain the behavior of the distribution, figure 6 shows some normalized patterns of $dN/d\tau$ which were directly calculated by using equation (10). According to our model, the integrated result becomes several forms of an exponential type, a power-law type or an exponential-with-a-tail type depending on the various conditions even when the individual SH has a reaction that follows the Poission process.

3.3. Parameters of the histories of Modern China and the Troubles and their interpretation
The τ-distribution in our model is mainly determined by the extent of oblivion, namely the values of k, τ_0 and γ , and the probability for an event to occur under the influence of the preceding event, g and h. In the case of Modern China (figure 1), the τ-distribution in the foreign affairs differs with interacting countries. The $dN/d\tau$ for Taiwan shows an exponential behavior that corresponds to the curve b in figure 6, so that the h and γ must be small. Hence in this case both countries of Modern China and Taiwan have passed time without forgetting their foreign policies and diplomatic relations in the past irrespective of their significance. The $dN/d\tau$ for USSR and Russia seems to obey a power-law in $\tau>9$ days, which corresponds to the case of small g and h (hence a small λ), and a large γ. This is the situation such that the memory scarcely remains between two countries though the occurrence of reactive event is retarded. On the other hand the $dN/d\tau$ for USA is an exponential-with-a-tail type so that the values of b, e and γ are all small, or h is small and γ is large. In the former case a reaction occurs with almost similar magnitude as the action with scarcely forgetting it, whereas in the latter case the probability for occurring the reaction is not so dependent on the magnitude of the shock SH

receives by an action, with scarcely retaining the memory.

In the case of Northern Ireland Troubles, the power of dN/dτ differs with the situation of the Troubles as seen in figure 4, the value of power being $-(k+1)$. Since the power is small in 1970s, peoples slowly forgot the events. The extent of oblivion became large although it fluctuated with time in 1980s to result in a much larger value in 1990s, indicating faster oblivion of the event with the longer continuation of the Troubles.

4. Conclusion

In the history of Modern China, the distributional feature of inter-event time has subtly differed with the different countries of foreign interaction, and in Northern Ireland the distribution has also changed with the stage of the Troubles. The τ-distribution as a whole, therefore, is just a superposition of many components with subtly different distributions from each other.

The occurrence of historical events is well noticed as the resultant that is participated from many stake holders. When we consider the cases of foreign affairs between two countries and the cases like the Troubles where friends and enemies are definitely divided, however, history can be interpreted as the repetition of the exchange of action and reaction between two substances that have different values from each other. Extending such an image to a general case, the occurrence of a historical event can be interpreted as the similar phenomenon to the exchange of information between two participants as the correspondence between two persons [12,13], the replying to a received e-mail [13,14], or the conflict around the articles in Wikipedia [15,16]. Since the historical event is just the resultant of the human activities, it is not so strange that its characteristics are similar to the other activities as such.

Appendix: Forms of the factors *G'* and *H'*

The mathematical expressions of *G'* and *H'* which appeared in equations (11) and (12) are respectively given by

$$G' = \left[2 - erfc\left\{ \frac{b}{2c^{1/2}} - \frac{b+e+h\eta\tau}{2c^{1/2}} \right\} \right] \cdot \exp\left\{ -\frac{bh\eta\tau}{2c} + \frac{(b+e+h\eta\tau)^2}{4c} \right\} \qquad (A1)$$

$$H' = 2 - erfc\left(\frac{b}{2c^{1/2}} - \frac{\alpha}{2} \right) + \frac{c^{1/2}(\alpha^2 - 2)}{b\alpha}\left\{ 2 - erfc\left(\frac{b - c^{1/2}\alpha}{2c^{1/2}} \right) \right\} - \frac{2c^{1/2}}{\pi^{1/2}b}\exp\left(\frac{(b - c^{1/2}\alpha)^2}{4c} \right) \qquad (A2)$$

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