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Physica A 391 (2012) 5978-5986

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Evolution of groups with a hierarchical structure

Teruaki Ohnishi

Institute of Science and Technology for Society, Urayasu, Chiba 2790012, Japan

ARTICLE INFO

Article history: Received 4 May 2012 Available online 18 June 2012

Keywords: Group evolution Dynamic behavior Hierarchical structure Power exchange Cellular automaton Agent model

1. Introduction

ABSTRACT

The universal occurrence of a hierarchical structure and its dynamic behavior in various types of group, living or abstract, are discussed. Here the word "group" refers not only to tangible aggregation but also to invisible aggregation of social psychological and of geopolitical meaning. The evolution of these groups is simulated using a model of agents distributed on the lattices of cellular grids. It is assumed that agents, fearing isolation, interact asymmetrically with each other with regard to exchange of "power". As an indicator of hierarchy, the Gini coefficient is introduced. Example calculations are made for the aggregation, fusion and fission of animal groups, and for the appearance of a powerful empire and the rise and fall of supremacy. It is shown that such abstract objects evolve with time in accordance with the universal rules of groups common to birds and fish.

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[1], it is natural to expect that the world will dynamically evolve with time. From the viewpoint of system dynamics, small and weak countries will tend to aggregate around some power center that is controlled and integrated politically, militarily, economically, or culturally by the center to form a supremacy. The supremacy itself, however, may be superseded by a new supremacy after (i) defeat in a war between supremacies, (ii) the occurrence of natural disasters such as earthquakes, volcanic eruption or climate change, (iii) the emergence of self-inconsistency in the supremacy, such as military expenditure that exceeds income, or (iv) a food crisis in the sense of Malthus due to population growth. In these cases, small and weak counties that were constituents of the previous supremacy will reshuffle and form a new group to integrate again in a new supremacy to form a new world order. When more than one supremacies exist at the same time, an exclusive and repulsive state may emerge among them. It has been pointed out that the rise and fall of supremacy has occurred almost cyclically throughout world history [2–5].

When we consider the world as a system consisting of many nations and autonomies that mutually influence one another

In areas surrounding the Mediterranean, many large and influential geopolitical supremacies have arisen since ancient times. Every supremacy subjugated surrounding small countries by force of arms and forced them to enter a group. Some small and weak countries voluntarily joined the group to secure profit and their own interests. Fig. 1 schematically shows a time plot of the extent of supremacies that appeared quasi-cyclically in the Mediterranean region from the 5th century BC to the 19th century AD. Portugal held real power in the world in the 16th century AD, Holland in the 17th century, England in the 18th and 19th centuries, and the USA in the 20th century.

Examples of the appearance of a collective phenomenon on the human scale include terrorist and political groups following a certain ideal [7,8], people who vote for a specific candidate in an election according to their opinions, and societies in which people adopt similar ideas and attitudes under some restrictive religious conditions. Fashion is also an example [9], whereby people imitate general trends in society or the people around them because they fear isolation or the disadvantages it may bring. In this case, information transfer first occurs among people in the fashion industry. Those who





E-mail address: ohnishi2015@yahoo.co.jp.

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Fig. 1. Transition of supremacy in the Mediterranean area. Territorial extent is taken here as a measure of power, which was roughly estimated using the DK Atlas of World History [6]. Crtg, the Cartago; Grc, Ancient Greece, Ptlmy, the Ptolemy Dynasty in Egypt; Roman, the Ancient Roman Empire; Byzant, the Byzantine Empire; SassPrs, the Sassanid Dynasty in Persia and its successors; Umayya, the Umayyad Dynasty and its successors; Frnk, the Frankish Kingdom; Hrom, the Holy Roman Empire and its ancestors; Spn, Spain; Eng, England; Ottoman, the Ottoman Empire.

acquire the information tend to imitate the fashion and transfer it to people around them who have yet become aware of the fashion. This results in transfer, infection and expansion of the fashion outwards [10,11]. In such a situation, the origin or center of the fashion acts as a social psychological attractor, so that people are attracted to the center, resulting in the spontaneous formation of a psychological group around the center. In this case the center of fashion is interpreted as a leader who leads people in a psychological sense. In general, however, since any fashion has a finite lifetime, the number of supporters gradually decreases after its heyday and its place is finally taken by another new fashion. The time variation for the extent of a fashion follows a wave whose peak corresponds to the heyday of the fashion [10]. As successive fashions change, the time behavior would more or less resemble that in Fig. 1, although the time scale would differ.

Apart from human-related phenomena, marked characteristics of aggregation clearly appear for the collective motion of small creatures such as insects and of vertebrate animals [12]. General characteristics observed for primates, other mammals and marine animals are leadership by a strong male or a senior individual and the successive formation of new groups via dynamic fusion and fragmentation of group members [13–15]. Antagonism has also been observed between groups with regard to securing territory and habitat segregation [13–15]. In addition to collective phenomena for large mammals, flocks, shoals and swarms are formed by many types of birds, fish and social insects such as ants and bees to move in a specific direction for a certain purpose. The migration of birds and fish seems to involve a few leaders who have far more information than the other individuals [16]. Ants and bees form swarms around a centralized individual known as the queen [17–20]. The unified movement of a swarm around a leader indicates the existence of information that all individuals possess in common. In fact, direct exchange of information [21]. The leader is the individual situated at the top of the hierarchy. In a larger group, however, direct exchange of information becomes impossible; it is transmitted via a local wave of communication propagating throughout the whole group. In such a case, the hierarchical structure does not form according to the amount of information to emerge.)

In what follows we consider a small group in which a single or a few leaders appear. The methodology used, which involves a cellular automaton model and the introduction of agents, is described in the next section. Simulations using this model are described in Section 4 regarding the formation of groups of living things and the appearance of abstract groups centered around a supreme power. Conclusions on the necessity of a group hierarchical structure for the survival of living things are presented in Section 4.

2. Agent model for a cellular field

The following common trends exist in animal groups and groups resulting from human activities:

- (1) A group is a system in which each constituent mutually interacts with surrounding individuals, usually exchanging information or some sort of power in the case of animals, and material, information and some sort of benefit and power through trade, finance and emigration in the case of a geopolitical system. The individual who possesses the maximum amount of these attributes becomes the leader of the group.
- (2) Individuals constituting the group are, in general, afraid of being alone for several reasons, so that they approach the leader socio-psychologically, geopolitically or physiologically. By contrast, the leader has a tendency to control the surrounding individuals to form a social order centering around himself.
- (3) Such a system is dynamic, so the supremacy rises and falls, and the group constituents meet with and part from each other.

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In what follows, we construct a unified model of a group. A constituent of the group is called a general agent or simply an agent, whereas the leader is called a power agent. Moreover, material and/or information exchanged among constituents is simply called power.

We consider a system of multiple agents moving on a two-dimensional cellular region of $(L \times L)$ in extent. Each agent corresponds to an individual or a constituent of the group considered. It is asumed that each agent mutually interacts with the other agents within a radius of r_c over a discrete time step Δt to result in acquisition of power. Interaction includes finance, international trade, tactics and persuasion in the field of politics and economics, observation of the behavior of others, disputes, immigration and exchange of culture in the case of human activity. For social animals and insects, interaction includes competition among the group, observation of the behavior of others, dance signals, transfer of crying, chirping ant other sounds as information, and the release and detection of pheromones. The discrete time Δt is an abstract time period within which one of the interaction processes described above takes place on average. The power of agent *i* at time $t + \Delta t$, $p_i^{t+\Delta t}$, is given by

$$p_i^{t+\Delta t} = p_i^t - \alpha \left(p_i^t \right)^m + \sum_j \Delta p_{j \to i}, \tag{1}$$

where α and *m* are constants. The second term on the right-hand side is the decrease in power during Δt , which is *m*-exponentially proportional to power, that is, it increases geometrically. The third term, $\Delta p_{i->i}$, is the power that agent *i* acquires through interaction with agent *j* and is given by [22]

$$\Delta p_{j \to i} = \delta / \left[1 + \exp\left\{ -\eta \left(P_i - P_j \right) \right\} \right], \tag{2}$$

where δ and η are constants, and p_n is the power of agent *n*. When $\eta = 0$, Eq. (2) gives the same power to two agents participating in the interaction, leading to an egalitarianism system. By contrast, when $\eta \gg 1$, the system gradually shift to a hierarchical society, since power is selectively invested in the agent with greater power.

Our model assumes that $n_p(=1)$ agents with power exceeding a threshold P_m become power agents. It is assumed that power agent M has the following attractive force towards general agent i, whereby the individual approaches the leader due to the fear of isolation:

$$f_{iM} = \varepsilon P_M \exp\left\{-\gamma \left| r_i - r_M \right| \right\},\tag{3}$$

where ε and γ are constants, r_n is the position of agent n, and P_M is the power of power agent M. Since the form of the attractive force is unimportant in our case, an exponential type was introduced here. Moreover, it is assumed that a repulsive force occurs between two power agents. The force felt by power agent M from power agent K is given by

$$F_{MK} = -EP_K \exp\left\{-\gamma \left| r_M - r_K \right| \right\},\tag{4}$$

where *E* is a constant parameter. For convenience, it is assumed that the extent of the force is given by the number of cells along which each agent moves in the direction of the force during Δt .

In addition to movement towards the force, each agent randomly moves to one of the vacant lattices of the Moor neighbor with every time step, which is a typical assumption for traditional models of this type [23–25]. When a power agent moves, if the new position is already occupied by a general agent, he replaces that agent. By contrast, a general agent is only allowed to move when the proposed new lattice position is vacant. For both power and general agents, a finite probability R is assigned to the decrease in power during period $[t, t + \Delta t]$. When agent *i* experiences a decrease in power at $t = t_1$ according to this probability, it is assumed that power decreases in $t > t_1$ by a constant factor α (< 1) in every time period Δt until it has decreased to less than its initial value. This assumption is introduced to imitate the finite lifetime of agents.

In the following calculation, n_a agents with randomly invested power in the range [0.9, 1.1] are randomly distributed at an initial time on a two-dimensional cellular plane with L = 100. The cyclic boundary condition is adopted, together with $r_c = L/(\pi n_p)^{1/2}$, $P_m = 400$, $\gamma = 2/L$, and $\alpha = 0.998$. Within time $[t, t + \Delta t]$, every agent successively interacts only once with the other $(n_a - 1)$ agents, so that the total number of mutual interaction is $n_a(n_a - 1)/2$.

3. Simulation of aggregation

First we investigate the feasibility of this model by mimicking the movement of a fish shoal. We assume the existence of one leader ($n_p = 1$) in a group of $n_a = 200$ constituents who crowd together around and then follow the leader. The leader moves at a velocity of v = 2.0 cells/ Δt along the edges of the system in an anticlockwise direction. When the leader disappears, a second power agent becomes the new leader and, after reaching the edge, follows the same movement along the edges as the first leader. This is reminiscent of a shoal of small fish moving in a water tank. In our model, a vague feature appears at t = 300 and a definite feature following the leader at t = 500, which lasts thereafter, as observed in Fig. 2. Since the period during which one power agent acts as a leader is finite, the leader is frequently replaced. Such a frequent change has been observed in real situations [26]. Plots for t = 800 and 1300 show the situation a short time after a change in leader, whereas t = 900 is the time at which the second power agent appears as a new leader after disappearance of the first leader. For this reason, the outer shape of the shoal at t = 900 becomes disarranged and all constituents are in a state of movement towards a new leader. Thus, when the leader frequently changes, the shoal features do not remain constant,



Fig. 2. Simulation of a fish shoal circulating in a closed container. The single power leader and general agents are denoted by red and blue dots, respectively. As shown by the red arrow in the top left plot, the leader moves along the edge of the region in an anticlockwise direction at velocity v = 2.0 cells/ Δt . The number shown in each plot represents time. $n_p = 3$, $n_a = 200$, $\varepsilon = E = 7.5 \times 10^{-3}$, and $\delta = 1.25 \times 10^{-3}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

even after a long time. The hierarchy regarding the quantity of power gradually increases to t = 4000, when the power of the leader P_{max} becomes 486 and the power of the weakest individual P_{min} is 1.25, with an almost linear distribution of power between them. After this, although the power of the leader hardly changes ($P_{\text{max}} = 470$ at t = 9000), P_{min} increases with time ($P_{\text{min}} = 55.6$ at t = 9000), indicating gradual relaxation of the hierarchy with time.

Fig. 3 shows a simulation of flying birds. The power agent flies in the upward direction (defined as the *y* direction) at velocity v = 3 cells/ Δt . A coordinate system is applied whereby the leader remains at position (*x*, *y*) = (50th cell from the left edge, 95th cell from the bottom edge). The general agents to move randomly at the initial time so that they never have a velocity component in common in the *y*-direction. Notwithstanding, the general agents show a tendency to follow the leader at t > 300 and their velocity in the *y*-direction gradually approaches that of the leader over time. Therefore, a clear trail feature as a flock appears at t = 500. As time further elapses, however, this trail feature gradually disappears and the flock finally becomes an aggregation of agents around a leader at t = 4000. The hierarchy in the system also increases with time up to t = 4000. The dynamics seems to closely mimic the real features observed for flying birds in nature.

Fig. 4 shows the evolution of agent distribution on a two-dimensional plane in a system with three power agents, each with a finite lifetime. The system features evolve quite dynamically because the power agents repel each other during attraction of general agents. Moreover, a new power agent appears at a different position after one power agent disappears, and the general agents tend to meet and part, depending on the distance from and strength of the power agents. Although qualitative, these plots seem to visually reproduce the features of territorial change exhibited by bird flocks and mammal groups [13–15], including changes in the number and identity of constituents. They are also reminiscent of certain collective



Fig. 3. Simulation of a flying bird flock. A leader (red dot) flies in the direction of the red arrow in the upper left figure at velocity v = 3.0 cells/ Δt . The number in each plot represents time, $n_p = 1$, $n_a = 200$, R = 0.0, $\varepsilon = E = 7.5 \times 10^{-3}$, and $\delta = 2.5 \times 10^{-3}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

human behaviors of humanity, such as the fusion and fragmentation of members in a matrilineal family [27] and the ups and downs of social anthropological groups in hierarchical Japanese society [28].

Next we investigate whether it is possible to explain abstract phenomena that collectively occur in relation to human activity. In the history of mankind, human groups such as villages, states and nations hardly remained permanently in isolation, but repeatedly exhibited fusion and fragmentation through alliance and coalition with other groups resulting from mutual interaction, such as combative conquests and cooperative defense. In our scenario, when a system becomes hierarchical through accumulation of material or information by only a few specified groups as a result of such mutual interactions, these groups become power agents and the result is the formation of large-scale supremacies. We take examples from modern Europe and the current world situation.

In Central Europe, many small nations, principalities and free cities appeared from the Middle Ages to the 19th century, each of which was independent. Frequent exchange of materials and information occurred among them. Unifying these autonomies, the German Federation arose in Central Europe in 1851 and the Italian Empire in the Italian Peninsula in 1871. According to our model, such unification of autonomies can be interpreted as the formation of a geopolitically formed group centered around the most powerful nation. In analyses of politics, the military, economics and cultural advancement, an index of the power of a supremacy is typically used. For convenience, we use population and GNP as quantitative measures of power. We assume that a hierarchical distribution of population and GNP arises as a result of discriminating interaction among the constituents.

Fig. 5(a) shows Lorenz curves for (i) the German Federation in 1855 [29] for the 18 constituent autonomies with the greatest power, (ii) the Italian Empire in 1871 [29], which comprised nine independent regions, and (iii) Europe in 1871 [30] for the 16 nations with the greatest power, all using population as a measure of power; and (iv) the world in 2006 [31] for the 50 countries with the greatest power using GNP as a measure of power. Gini coefficients for the distribution of power were calculated as (i) 0.67, (ii) 0.20, (iii) 0.50, and (iv) 0.64 from these Lorenz curves. The Gini coefficient is an index indicating the homogeneity of a distribution. A value of 0.0 corresponds to a completely equal society and 1.0 to an extremely hierarchical society in which only one agent accumulates power. Fig. 5(b) shows Lorenz curves for $n_a = 50$ in our model; these give almost the same Gini coefficient values as in (a). For Europe in 1871, our calculation does not necessarily reproduce the observation. This indicates the possibility that interaction between strong and weak nations in Europe was not sufficient at that time; moreover, Europe in the mid-19th century seems to have been overwhelmed by a small number of highly powerful nations. Apart from Europe in this case, our model reproduces the real world well, indicating that quantitative parameters such as population and GNP can be used as measures of power for groups of nations, or conversely that the hierarchical population and GNP distributions in the real world are manifestations of discriminating interactions among groups. Moreover, the data possibly demonstrate that the group with the largest population or greatest GNP can become a nucleus for supremacy at that point in time.



Fig. 4. Features of group fusion and partition when three power leaders (red dots) exist in the system. The number in each plot represents time. $n_p = 3$, R = 0.001, and the other parameters take the same values as in Fig. 3. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 5c–f shows the agent distribution on a cellular plane representing geopolitics. The dispersion or aggregation of agents indicates the strength of coalition among them. The plots show that the extent of the coalition in the German Federation was weak, whereas it was quite tight in the Italian Empire. We have no definitive evidence, however, indicating whether the agents in Fig. 5c–f actually reveal the relative position in a geopolitical sense and the tightness of coalition among the constituents. However, we know that the German Federation was a weak coalition of autonomies forced to aggregate from above, and that early disintegration of this federation in 1866 was followed by a civil war between Prussia and Austria. The German Empire born in 1871 was a unified nation founded by force [4]. Thus, we can guess that a weak coalition of states occurred in Germany in the mid-19th century. By contrast, in the case of Italian unification in 1871, small states centered around the Sardinian kingdom voluntarily opted for unification and amalgamation with Italy via a poll of their citizens, so it is reasonable to imagine a tight coalition among the states.

We present another example demonstrating that a geopolitical system can be modeled by analogy to groups. In the world system considered here, each agent corresponds to an aggregation of people such as a village, state or nation, and a power agent to a strong aggregation of people, situated at the top of a hierarchy. When such agents come together to form a centralized supremacy, the power of the supremacy Σ is calculated as the sum of the power for the power agents and the power emanating from general agents gathered around the power agent.

We assume that three power agents exist. Fig. 6(a) shows the time evolution of their power. A decrease in power after a randomly determined but finite lifetime occurs and a new power agent appears after disappearance of the preceding agent. Fig. 6(b) shows the time evolution of the power Σ of supremacies successively formed centered around the power agents (cf. Fig. 1). Although the Σ curves are not smooth as in Fig. 1, but fluctuate randomly over time as general agents move over and back cross the influencing boundary ($r = r_c$) of the power agent, we can say that the general feature of disappearance of a supremacy continually followed by a new supremacy reproduces real-world situations to a large extent.

Fig. 7 shows the features of a system, derived with the same parameter values as in Fig. 6, for the sudden appearance of a negative nucleus, that is, a power agent evaded by general agents. In this case the force exerted by the power agent is repulsive. Fig. 7(a) represents a situation in which a predator suddenly appears in a bird flock or fish shoal. The same movement as in Fig. 2 is assumed for the power agent, who moves along the edge of the system in an anticlockwise direction

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Fig. 5. (a,b) Lorenz curves for the German Federation in 1855, the Italian Empire in 1871, Europe in 1871 and the present world in 2006, and (c-f) the distributional state of agents on an abstract geopolitical plane. Lorenz curves show the relative accumulation of the power as a function of the relative number of agents. GNP was used as the measure of power for the present world, whereas population was used for the other cases. (a) Real observations and (b) model results at t = 20000 simulated using $n_a = 50$, $n_p = 1$, $\varepsilon = 3.0 \times 10^{-3}$, $E = 3.0 \times 10^{-2}$, $\delta = 8.0 \times 10^{-2}$, and $\eta = 10.0$. (c) The German Federation in 1855, (d) the Italian Empire in 1871, (e) Europe in 1871, and (f) the world in 2006.

at a velocity of 2 cells/ Δt . Because of the cyclic boundary assumed for the model, it is also tacitly assumed that predators exist cyclically at the top and bottom and right and left sides of the system. The general agents (the prey) are therefore in a state surrounded by a group of predators. Thus, the prey cannot escape from the force field generated by the predators so they aggregate spherically around the center [32] (although in our case they aggregate in a rather square form corresponding to the square movement of the predator).

Fig. 7(b) shows disorder of the system involving the movement of general agents when a negative input appears at the center at t = 0. This is reminiscent of a stampede in response to a disturbance, such as a stone thrown into a calm shoal of fish. In this case the general agents promptly move outwards from the disturbance to evade danger and a circular density wave of agents is observed. Such concrete examples demonstrate that the model (Fig. 6) also describes the dynamics of animal groups. This strongly indicates that the behaviors of both world systems and animal groups are merely different manifestations of the same phenomenon, that is, the phenomenon of aggregation.

4. Conclusion

As observed in the previous section, resulting from nonlinear internal interaction, a mass becomes a group with one or more leaders if a hierarchical structure regarding the distribution of material and/or information occurs. Individuals that can

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Fig. 6. Time evolution of a system with three power leaders and 1000 general agents. (a) Changes in leader power. $\varepsilon = 1.5 \times 10^{-3}$, $E = 2.5 \times 10^{-4}$, and $\delta = 1.0 \times 10^{-4}$. (b) Time evolution of the extent of power Σ of supremacies formed around power leaders. For simplicity, only supremacies with the greatest power are shown around t = 10000 and 20000.



Fig. 7. Dynamics of predator (red dots) and prey (blue dots) behavior derived using the same parameter values as in Fig. 6. (a) Behavior when the predator moves along the edge of the system in an anticlockwise direction, indicated by a red arrow. (b) Behavior when a disturbance such as a predator or a stone is suddenly input at the center of the system. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

form such groups are widely distributed and range from social insects, animals and humans, to states, nations and sociopsychological groups. Although group existence is universal, the rules for forming a group are quite simple and general, irrespective of the various scales of space and time involved. Group formation seems to arise from the fundamental instinct of defense for a creatures survival. We can say, therefore, that the formation of a hierarchy in a group is an important and instinctive trait for animals, including humans, to acquire an advantage for species survival. A society in which a hierarchical structure is realized seems to be more stable and normal than a society without it. Thus, a hierarchical structure seems to be a survival mechanism, irrespective of whether the driving forces are conflictive or peaceful. Supremacy has appeared and disappeared throughout the course of human history. When we consider that such supremacy is the manifestation of a

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hierarchical structure among groups of states and nations gathered around a powerful nation with a centralized force, the rise and fall of supremacy is an indispensable phenomenon for human survival to date and into the future.

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