POSSIBLE POWER LAW DISTRIBUTION OF HISTORICAL INTEREVENT TIME

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Frequency distributions were investigated regarding the intervals between historical events and a model was proposed to explain their appearance. The interval τ of the times between two successive events of wars, conflicts and their related incidents was studied covering ten and several historical subjects. Many of them were found to show a power-law distribution with a power -1~ -2. A complex society consisted from many stakeholders and many incident items were considered in modeling the situation, where a prerequisite was introduced such that each stakeholder can induce events of various types of incident items as reactions to an initially occurred event as an impact. The occurrence rate of such reactive events was also assumed to nonlinearly depend on the strength of the memory of the impact, of which the stakeholder was oblivious nonlinearly with time. This model was found to well explain the power law together with the stretched exponential type distributions of the interevent time.

Keywords: historical event; interevent time; power law distribution; memory; oblivion; occurrence rate.

1. Introduction

When a certain natural phenomenon is an independent event on the environment and its occurrence rate is random in time, it is a Poisson process. In this case, the probability distribution of the frequency regarding the intervals or the interevent times between two successive events becomes exponential. However, there exist many examples in nature where the interevent time follows a power law, not an exponential law. In such cases, they are considered to suffer a change in their random processes caused by the intervention of some specific perturbation.[1-5] In the phenomena in which the human being participate, there necessarily appear the individual will and/or the intrinsic characters as the human being such as the execution of action with a deterministic purpose, so that we can not easily imagine for any event to occur randomly in time. In this case, the frequency distribution of the interevent time becomes, in general, a power law type or the stretched exponential type. Many examples have been pointed out, which show the power law distribution on the interevent time such as the e-mail sending, [6,7] the surface mail sending,[8] the printing behavior of computers,[9] the Web browsing, [10-14] the library loan,[15] the movie watching,[16] the twitter,[17] the book sale rankings[18] and so on.

Noticing the characteristics of those phenomena being the repetition of a long stationary period and a concentrated burst of events and examining the correlated behavior between events, Goh et al.[19] has pointed out a possibility for the distribution to be understandable in terms of the memory. By using a model such that the memory of a person affect his/her successive action, Vazquez [20] has tried to explain the power law distribution of information sending, whereas Sano et al.[17] have studied the distributional feature of the access number to a blogoshere. Han et al.[12] and Vazquez et al [6,15,21] have respectively proposed the interest-based model and the decision based queuing model to explain the power law distribution. Zhou et al.[16] have pointed out that in the interest-driven system such as the rating movie the power law distribution appears not only in a collective, macroscopic level but in a private, microscopic level. While on the other hand Hidalgo[22] has reported the outcome of the power law appearance in a collective level by superposing many Poisson distributions.

Those former studies have discussed the interevent time regarding a certain unique item such as the e-mail sending or the surface mail sending by the selected people with the similar characteristics to each other. Our question is therefore whether or not the power law distribution on the interevent time is still held even for the so-called dirty samples such that the people concerned are belonging to the various classes of people with various values and various standards to decision. To investigate such a subject we treat the distribution of the interevent time regarding historical events, where we use chronological tables as our materials.

In Section 2 some distributions are exemplified which are deduced from some chronologies. In Section 3 to explain the distributions of the power law type and the stretched exponential type we propose a model such that an event successive to the former one arises with a rate depending on the strength of the impact given by the former event and on the extent of the memory preserved up to that time in a complex system composed of many parties or persons concerned (hereafter called as stakeholders). Concluding remarks are given in Section 4.

2. Frequency Distribution of the Intervals between Two Successive Historical Events

We call hereafter the classified sorts of events in history as the incident item or the item, which are for instance the domestic administration, the diplomacy, the economy, the religion or the culture in the history under consideration. When the events are derived randomly in time by a certain independent subject, the frequency distribution of the interevent time τ becomes to be an exponential type as already mentioned.

Fig.1(a) is the frequency distribution of τ for 2296 politics related events in the modern China occurred during 1930~2003.[23] In the following figures including this, the width of the bin on the abscissa and the frequency in that bin are adjusted so as to be a finite frequency in all bins from $\tau = \tau_{min}$ to τ_{max}, where $\tau = [\tau_{min}, \tau_{max}]$. Fig.1(a) is of an approximately exponential type or seems to be a superposition of, for instance, two types of exponential distributions, indicating that those events have occurred almost independently to each other.

In what follows we limit to the item where we can think of a certain event as an action and its subsequent event as a reaction to it, considering the relation of cause and effect between the events. As examples for such items, we here consider wars, conflicts and the history of countries under strained atmospheres.

Fig.1(b) shows the distribution of 402 interevent times of Western Wars taken place during 413~1881 AD,[24] which is of an exponential type. This indicates the random occurrence of wars with time as a Poisson process without any relation between the wars to each other. Although Richardson[25] has estimated the existence of 5 and 15 years cycles of wars in the period 1500~1931, he has also pointed out the possibility of the annual occurrence of wars during that period to be a Poisson distribution. Fig.1(b) is not inconsistent with Richardson's view that the occurrence of wars is a Poisson process.

Fig.(c) is the distribution of 47 time intervals of the major battles in the Korean War during June 25, 1950 and July 27, 1953.[26] Though it is not necessarily reliable from a statistical viewpoint due to the small number of events, it also seems to be exponential. Since in general there exist no communications between two parties concerned in wars, it seems possible that the offence in the Korean War had been practiced according to an independent operation on the foe. If this really is the case, then, we can understand the exponential distribution of τ, because the offence becomes to be the Poisson process.

Fig.(d) is the distribution of 232 terrorism related events occurred in Israel during 2001~2004, whereas Fig.(e) are the same ones of 259 event in USA during 2001~2004, and of 228 events in Iraq during 2002~2004.[27] Although they do not necessarily show smooth behaviors with τ, they can be said generally to follow the power law. The powers which make the Kolmogorov-Smirnov statistic[28] minimum are -1.13, -0.91 and -1.97 for the cases of Israel, USA and Iraq, respectively, and their average is -1.34. The probability distribution of worldwide victims by the terrorism during 1968~2006 follows a power law with a power -1.4.[28] At least with regard to the terrorism, therefore, both its extent and its interevent time become to follow the power law.

The substances of the terrorism are considered usually to make the fusion and fragmentation.[29,30] When we think of the whole terrorists in one country as one aggregate, we can interpret the successive terrorisms as a result of the repeated actions and reactions manifesting between such an aggregate and its anti-aggregate. Such an example is considered as a simple fundamental process from which we may possibly construct a model for the distribution of τ. Generally speaking, however, there exist so many stakeholders participating in the occurrence of events. We should note that the successive events appeared in history are the result of a quite complex system where an event of a certain item induced by one stakeholder becomes to be a cause which in turn induces another item by another stakeholder.

Fig.1(f) shows the distributions of 89 interevent times for the Crusades and the Military Religious Order during 1080~1571,[31] and 116 interevent times for the Crusades during 1071~1336,[32,33] whereas Fig.1(g) is of 93 intervals for the Crusade related events viewed from the Arabs[34] during 1096~1291. Although the numbers of events are not necessarily satisfactory from a statistical viewpoint, those distributions seem to follow the power law with the powers (d) -1.40 and -1.87, and (e) -1.47, respectively. The data corresponding to those figures (f) and (g) both range over 200 years so that the change of characteristics might take place in the system of stakeholders during such a long time span. This may be a reason why the distributions (f) and (g) seem to be made from multiple components.

Fig.1(h) is the distribution of 217 intervals for the overall events occurred in USA around the period of the American Civil War during 11 years from 1859 to 1869,[35] and Fig.1(i) is the distribution of τ for 560 events regarding the Northern Ireland Dispute reported during 1969~1996.[36] On the other hand Fig.1(j) shows the distributions of the τ for 476 events during 1970~2002 (blue [37]), and 211 events during 1950~1996 (red [38]) both for the Afghanistan Dispute, whereas Fig.1(k) shows the distributions for 132 events during 1961~1992 (green [39]), 342 events during 1961~1975 (red [40]), and 1093 events during 1961~1975 (blue [41]) all for the Vietnam War. The powers are respectively (h)-1.43, (i)-1.47, (j)-1.46 and -1.80, and (k)-1.11, -1.63 and -2.30. Figs.(i) and (j) show to what extent the distribution differs with different sources to be used. Those sources in (h)~(g) deal with a relatively wide range of items including the events around wars and conflicts, such as the background, the politics and the international relations, so that we should note that the events included do not always correspond to the simple repetition of actions and reactions between friends and foes as in (d) and (e).

Furthermore we show some examples for the cases, not under wars or conflicts, but under a certain strained condition somewhat similar to the war. Fig.1(l) is of the 530 interevent times for the general items occurred in Yugoslavia during 1941~1993.[42,43] Those include the events mainly of political changes together with the occurrences around the solidification of the country, the domestic administration, the diplomacy, military affairs and its fragmentation. Many events in Yugoslavia in that period had occurred as the results of the power struggle or the calculation between many stakeholders so that their arena had been quite complex.

Fig.1(m) shows the distribution of 254 events regarding the interactive relation between China and Soviet Union and Russia during 1941~2008.[44] These are the set concerning the diplomacy between the two countries and its relating subjects. It shows a power law distribution with a power -1.08. On the other hand, Fig.1(n) is for the 503 time intervals of events occurred between China and Taiwan during the same period as (m).[44] This is a distribution, not of an exponential type nor a power law type, but of the so-called stretched exponential type of the form $X^{A}exp(-BX)$ with the power A=-0.49. The frequency distribution of the interval time between two successive internal disturbances or domestic wars in China since 14th century has been revealed also to follow a stretched exponential type.[45]

Lastly Fig.1(o) shows the distribution of 380 events regarding the diplomacy between China and USA deduced from the same source as (m) and (n),[44] which is approximately of an exponential type. Those figures (m)~(o) mean that the interevent times with regard to China have different distributions from opposite nation to nation, indicating Chinese politics of diplomacy had differed with nations with which China makes interactions. Generally speaking it is required a long time to construct a stable and reliable relation of diplomacy between two countries. The power law distribution appears in nature only when the environmental condition around its phenomenon becomes to the self-organized criticality.[46] If the interpretation as such can be allowed also in the human history, we can say that the diplomatic relations between China and Taiwan, and China and USA had not been in mature, but in a sub-critical transition state in that period.

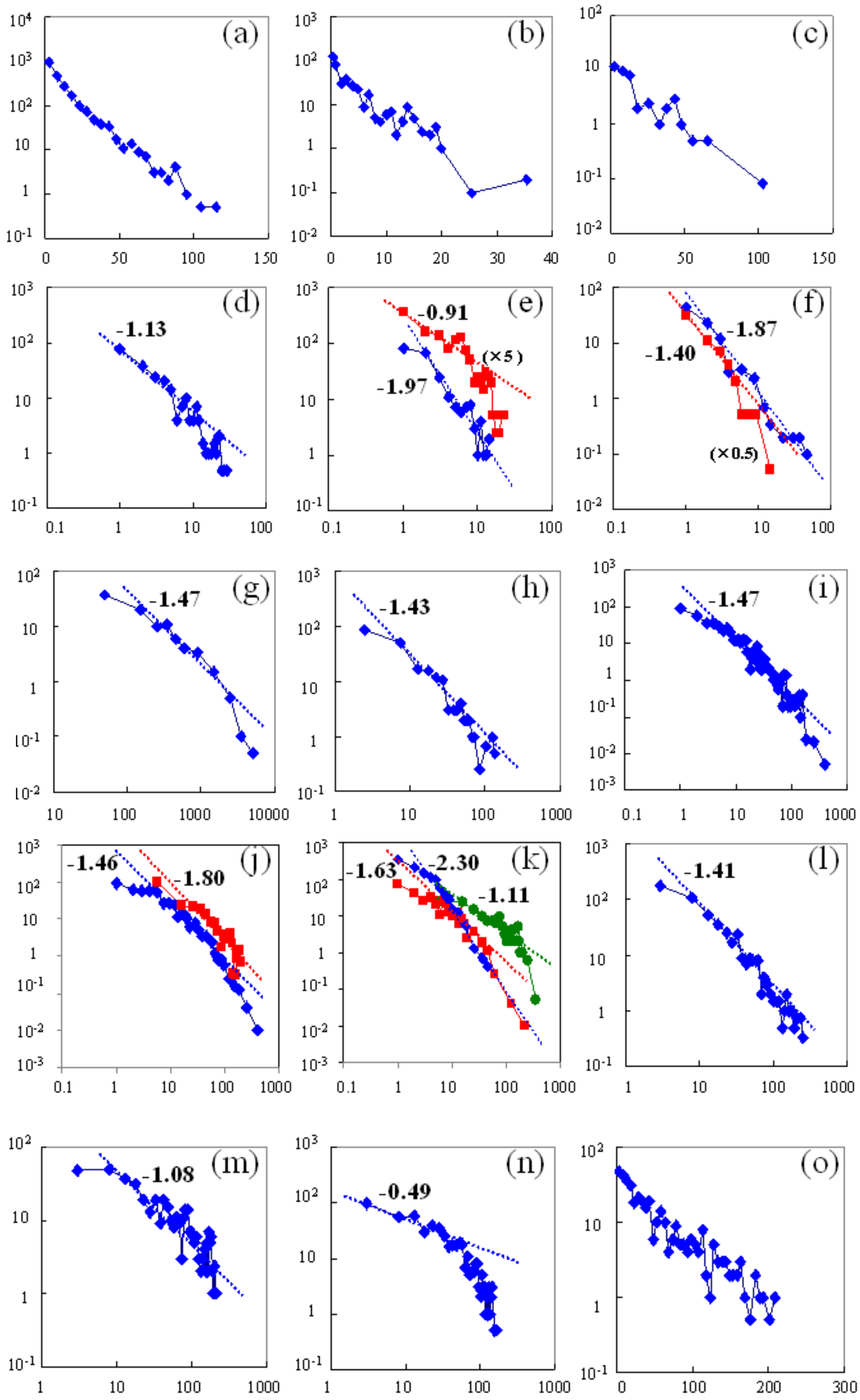

Fig.1 Probability distributions of the interevent time regarding historical events. The abscissa is in a linear scale only in (a)~(c) and (o), otherwise in a logarithmic scale. The unit of the abscissa is in years only for the cases of (b) and (f), otherwise in days. The dotted lines and the numerals are the power law lines which make the Kolmogorov-Smirnov statistic minimum and their powers, respectively. (a) modern China politics, (b) wars in the world, (c) the Korean War, (d) terrorism in Israel, (e) terrorism in USA (red) and Iraq (blue), (f) the Crusades (red,[31] blue,[32,33]), (g) the Crusades from the Arabs, (h) USA in the era of the American Civil War, (i) the Northern Ireland Dispute, (j) the Afghanistan Dispute (red,[38] blue[37]), (k) the Vietnam War (red,[40] blue,[41] green[39]), (l) Yugoslavia, (m) China-Soviet Union and Russia relation, (n) China-Taiwan relation, and (o) China-USA relation.

3. Oblivion Model of Memory in a Complex System

When we characterize the events which have taken place in the society, we have the following image for its modeling.

(1) There are so many incident items intermingled in the society. In the region of each item, social events have appeared one after another to result in the formation of chronology. Moreover in the society there are also many or almost infinite stakeholders which interact with each item. A new event belonging to a certain item may once come to appear as a result of, for instance, the power relations or the interests between the stakeholders. The event thus newly occurred has a finite impact for all stakeholders, but the extent of its importance felt by the stakeholders differs with each stakeholder. When the time interval becomes long for the successive occurrence of events, the attenuation of memory or the oblivion effect can not be avoided. Namely, the impact and the importance of the event already occurred attenuate with time and its social meaning also changes with time in the memory of stakeholders. The next event which successively occurs as a reaction to the preceding event, therefore, may be changed in its character so that it may possibly appear as an event in a quite different item from the initial one. In this case the extent of the social impact of the reaction may also differ from that of the initial event. The newly born event thus occurred becomes an action for the next stage, and such a process is reiterated again in the society.

The editors of the chronology or the chronological table have only listed such events in the order of the occurrence with scarcely including information on the extent of social impact. In a previous section we have used such chronological tables as sources with regard to which there exist some problems as follows.

(2) It depends on the editor's subject to determine which of the events historically recorded is to be adopted as suitable for the chronological items. He/she may try to simplify the table by the omission or the combination of events which are close in time, or homogenize the space usable per unit time span by limiting the number of events. In editing the table, therefore, it can not be avoided the oversight of the successive events which take place during a short time span so that their appearance rate may be estimated too small. If we introduce a compensation factor for such an artificial effect of segregation, we may think of something like the dead time correction or the counting efficiency in counting too strong radiation.[47] In what follows, however, we consider our model without any such correction factors.

We consider here the society consisted from S stakeholders and E incident items. An event belonging to the item e' ($\leqq E$) is assumed to occur at a time t. We also assume the random occurrence of events in time, following the Poisson process. Namely the number of events $\zeta(\tau)_{se}\delta\tau$ which occur during the time $(t+\tau, t+\tau+\delta\tau)$ at a time τ after t is given by

$$\varsigma_{se}(\tau)\,\delta\tau = a\lambda_{se}\cdot\exp(-\lambda_{se}\tau)\cdot\delta\,\tau \tag{1}$$

where a is a constant, and λ_{se} is the occurrence rate of the event belonging to e which is caused by the stakeholder s ($\leq S$) at $t+\tau$. We consider the event as a reaction to the event belonging to e', that is an initial action at the time t.

The impact strength $I'(t)$ which the s feels for the event e' at t must differs with each stakeholder; $I'(t) \equiv I'_{se'}(t)$. For this impact $I'_{se'}(t)$ as a received input, the s will make a reaction of the item e with a strength $I'_{se}(t)$. The quantities I' and I'' are, in general, different from each other, the events becoming escalated [48] when $I''>I'$ and settled down when $I''<I'$. We suppose the function which gives the relation between the input $I'_{se'}$ and the output I''_{se} as $\varphi(I''_{se}; I'_{se'})$. We have no information on the form of $\varphi(I''_{se}; I'_{se'})$. But if that relation follows the truncated Gauss distribution, for instance, the function φ, which corresponds to a sort of a mapping function from I' to I, may be given by

$$\phi(I''_{se}; I'_{se'}) = b_{see'} \exp\left\{-\varepsilon_{see'}(I'_{se'} - I''_{se})^2\right\}, \quad I'_{se'}, I''_{se} \in [0,1] \tag{2}$$

where $b_{see'}$ and $\varepsilon_{see'}$ are constants which depend on s, e and e', and we assume the quantities $I'_{se'}$ and I''_{se} normalized such as their maxima being unity. Although we have shown the form (2) as an example, its precise form is not required for the following discussion.

The quantity I''_{se}, which is the strength of the intension for s to cause an event e at t, will gradually decrease with time to become a quantity I_{se} at $t+\tau$, which is now different from I''_{se}. This is mainly due to the oblivion of memory,[49,50] which occurs nonlinearly with time, though the attenuation of the strength may be caused also by the change of social environment. When we presume the nonlinearity of the oblivion being inversely proportional to the powers of time, the I_{se} is given by

$$I_{se}(\tau) = \tau_0^{\alpha} \; I''_{se} \cdot (\tau_0 + \tau)^{-\alpha} \tag{3}$$

where τ_0 and α are constants.

The stronger the intention $I_{se}(\tau)$, the higher the rate for the s to cause the event e subsequent to the e' becomes. In reality in the case of the Vietnam War, immediate counterattacks were always made when the intense damage was sustained by the enemy attack, but it was not the case when the damage was not so severe.[40] This indicates the realization of the high occurrence rate of events when $I_{se}(\tau)$ is large, since the strong intention I''_{se}, which finally grows to $I_{se}(\tau)$ after the time τ, generally results from a strong impact $I'_{se'}$. Namely, the quantity λ_{se} may be given by some increasing function of $I_{se}(\tau)$, so that we may be allowed to assume the following form.

$$\lambda_{se} = d\left(I_{se}(\tau)\right)^{\beta} = C(I''_{se})^{\beta} \cdot (\tau_0+\tau)^{-\alpha\beta} \tag{4}$$

where d, C and β are constants.

The occurrence frequency of the event belonging to the item e, N_e, with respect to the interevent time τ is given by

$$\begin{aligned} \frac{dN_e}{d\tau} &= \sum_s \sum_{e'} \int_0^1\!\!\int_0^1 \phi(I''_{se} : I'_{se'})\; \varsigma_{se}(\tau)\; \psi(I'_{se'})\; dI''\, dI' \\ &= C(\tau_0+\tau)^{-\alpha\beta} \sum_s \sum_{e'} \int_0^1\!\!\int_0^1 \Big[\phi(I''_{se} : I'_{se'})\; (I''_{se})^{\beta} \times \\ &\qquad \times \exp\left\{-C(I''_{se})^{\beta}(\tau_0+\tau)^{-\alpha\beta}\,\tau\right\} \psi(I'_{se'})\Big]\, dI''\, dI' \end{aligned} \tag{5}$$

Here the quantity $\Psi(I'_{se'})$ represents the number of the event of *e'*, which occur at an arbitrary time *t* with the strength $I'_{se'}$. In the case of the approach of our process to an equilibrium condition after a long time, the quantity $\Psi(I'_{se'})$ becomes unrelated to the time *t*. Paying attention to the factors of the power of τ and making a zero'*th* approximation on the right-hand side of Eq.(5), we obtain

$$\left.\begin{aligned}\frac{dN_e}{d\tau} &\approx D_e\,(\tau_0+\tau)^{-\alpha\beta}\exp\left\{-C\left(<I>_e\right)^{\beta}(\tau_0+\tau)^{-\alpha\beta}\,\tau\right\}\\ &\approx D_e\,\tau^{-\alpha\beta}\exp\left\{-C\left(<I>_e\right)^{\beta}\tau^{1-\alpha\beta}\right\},\quad when\ \ \tau>>0,\ \alpha\beta<1\\ &\approx D_e\,\tau^{-\alpha\beta},\quad when\ \ \tau>>0,\ \alpha\beta\geq 1\\ &\approx E_e\,,\quad \tau\to 0\end{aligned}\right\}\qquad (6)$$

Here D_e and E_e are constants, and $<I>_e$ represents the average of the *I* for the event of the item *e* with respect to the quantities *I'*, *I"*, *e'* and *s*. Eq.(6) shows that the occurrence frequency of the event per unit τ becomes constant in the region of small τ, whereas in the region of larger τ, it distributes in a power law form with a power $-\alpha\beta$ when $\alpha\beta\geq1$, and a stretched exponential form when $\alpha\beta<1$. Which form it takes depends on both the extent of the oblivion of memory and the extent of the dependency for the promptitude of the reaction on the impact strength of an action. This means that the necessity of a prompt reaction decreases with time in stakeholders since the importance of the initial impact fades away and its meaning changes gradually with time,

4. Concluding Remarks

There usually exist multiple stakeholders for the event which takes place in a real society no matter how the society is simple. In the case of two stakeholders without any communication with each other, the occurrence of events becomes to be the Poisson process. When two stakeholders do the action and the reaction over again with respect to a certain item, it corresponds to a fundamental process of our model. The interevent times of such a process seem to follow a power law as seen in the examples of terrorism Figs.1(d) and (e).

Each stakeholder, in general, will cause the event of the same item as or of the different item from the initial one by deciding from the viewpoint of his/her own. If a certain event really occurs as a result of his/her decision, then the event becomes to be the next one which occurs under the influence of the preceding event. Since the newly occurred event makes the environment different from the previous one, the above process will be reshuffled and repeated again. When such a comprehension really is the case, history can be said to be a Markov process. As a result of the repetition of such a Markov process during a long historical time, therefore, a statistical trend emerges in the ensemble of the interevent time of events.

In the system with more than two stakeholders, the prediction is not possible, in general, which of the items by which of the stakeholders will appear in the next time no matter what the initial event is, so that our society is a sort of complex systems. For our society to work as a real complex system, information is required to propagate throughout the stakeholders without delay. In the case when that requirement is not satisfied, the above prerequisite as the Markov process can not also be satisfied and, as a result of it, fluctuations will appear in the probability distribution curves of the interevent time.

The powers of our examples cited above roughly range -1~ -2 around an average -1.4 ~ -1.5. This may be a sort of universality. If this really is the case, then, it indicates that our society may possibly be in a state of self-organized criticality under which various types of items appear according to the same dynamical law. This is reminiscent to the fact that the wave number of eddies along the turbulent flow follow the -5/3 power law no matter what the Reynolds number is. From the analogy of eddies, human historical events can be understood as to be gradually changed with time in cascades or in avalanches from a large and strong event to small and weak events as if they are something like a sort of eddies. In this sense, the social events adopted in this paper are in a state of turbulence in their social evolution.

In conclusion we constructed a model for the social events which appear according to the strength of the impact initially received and the extent of oblivion of the memory of that impact. When we assume a nonlinear form of Eq.(3) for the oblivion of memory and Eq.(4) for the occurrence rate of a new event, we can show the frequency distribution of the interevent time which follows the power law or the stretched exponential law no matter what complex the society concerned is. Further works remain to be done, however, to determine the way for selecting the sources, to strictly define the times for the beginning and the end of the history considering and to determine the appropriate events corresponding to the item considering together with the statistical confidence level of the value of power exponent.

References

1. G.M.Viswanathan, V.Afanasyev, S.V.Buldyrev, E.J.Murphy, P.A.Prince, H.E.Stanley, Nature **381**, 413 (1996).
2. S.Zapperi, A.Vespignani, H.E.Stanley, Nature **388**, 658 (1997).
3. B.Suki, A.-L.Barabasi, Z.Hantos, F.Petak, H.E.Stanley, Nature **368**, 615(1994).
4. P.Bak, K.Christensen, L.Danon, T.Scanlon, Phys.Rev.Lett. **88**, 178501(2002).
5. A.Corral, Phys.Rev. **E68**, 035102(R) (2003).
6. A.-L.Barabasi, Nature **435**, 207(2005).
7. J.-P.Eckmann, E.Moses, D.Sergi, Proc.Natl.Acad.Sci.USA **101**, 14333 (2004).
8. J.G..Oliveira, A.-L.Barabasi, Nature **437**, 1251 (2005).
9. U.Harder, M.Paczuski, Physica **A361**, 329 (2006).
10. A.Johanson, Physica **A296**, 539 (2001).
11. Z.Dezso, E.Almaas, A.Lukacs, B.Racz, I.Szakadat, A.-L.Barabasi, Physical Review **E73**, 066132(2006).
12. X.-P.Han, T.Zhou, B.-H.Wang, New J. Phys. **10**, 073010(1998).

13. A.Johansen, Physica **A338**, 286 (2004).
14. R.Crane, D.Sornette, Proc.Natl.Acad.Sci.USA **105**, 15649 (2008).
15. A.Vazquez, J.G.Oliveria, Z.Dezso, K.-I.Goh, I.Kondor, A.-L.Barabasi, Physical Review **E73**, 036127(2006).
16. T.Zhou, H.A.T.Kiet, B.J.Kim, B.-H.Wang, P.Holme, EPL **82**, 28002 (2008).
17. Y.Sano, K.Yamada, H.Watanabe, H.Takayasu, M.Takayasu, Phys.Rev. **E87**, 012805 (2013).
18. D.Sornette, F.Deschatres, T.Gilbert, Y.Ageon, Phys.Rev.Lett. **93**, 228701 (2004).
19. K.-I.Goh, A.-L.Barabasi, EPL **81**, 48002(2008).
20. A.Vazquez, Physica **A372**, 747 (2007).
21. A.Vaquez, Phys.Rev.Lett. **95**, 248701 (2005).
22. C.A.R.Hidalgo, Physica **A369**, 877 (2006).
23. R.Iechika, *Enlarged Edition: A Chronological Table of Politics in Modern and Present China* (Kouyou Shobou, Kyoto, 2004) (in Japanese).
24. Y.Nakamura, *A History of World Wars: A Separate Volume* (Hara Shobou, Tokyo, 1985) (in Japanese).
25. L.F.Richardson, *Statistics of Deadly Quarrels* (Pacific Grove, California, 1960).
26. B.Hannings, *The Korean War-an Exhaustive Chronology, Volume 3* (McFarland & Company Inc. Pub., Jefferson, North Carolina, 2007).
27. E.F.Mickolus, S.L.Simmons, *Terrorism 2002-2004, Volume 3* (Praeger Security international, Westport, Connecticut, 2006).
28. A.Clauset, C.R.Shalizi, M.E.J.Newman, SIAM Rev. **51**, 661 (2009).
29. N.F.Johnson, M.Spagat, J.A.Restrepo, O.Becerra, J.C.Bohorquez, N.Suarez, E.M.Resrepo, R.Zarama, “Universal patterns underlying ongoing wars and terrorism”, http://arxiv.org/abs/physics/0605035.
30. A.Clauset, K.S.Gleditsch, PLOS One **7**(11), e48633 (2012).
31. T.Hashiguchi, *Military Religious Order* (Kondou Shuppansha, Tokyo, 1971) (in Japanese), and *Military Religious Order of the Crusades* (Koudansha, Tokyo, 1994) (in Japanese).
32. G.Tate, *L’Orient des Croisade* (Gallimard, 1991).
33. A.Jotischky, *Crusading and the Crusader States* (Pearson Education Ltd., London, 2004).
34. A.Maalouf, *Les Croisades Vues par les Arabes* (Jean-Claude Lattes, 1983).
35. J.Nakamura, *Chronological Dictionary to Read American History: 19th Century* (Hara Shobou, Tokyo, 2011) (in Japanese).
36. T.Horikoshi, *A History of Northern Ireland Dispute* (Ronsousha, Tokyo, 1996) (in Japanese).
37. K.Maeda, S.Yamane, *A History of Afghanistan* (Kawade Shoboushinsha, Tokyo, 2002) (in Japanese).
38. L.W.Adamec, *Dictionary of Afghan War, Revolution, and Insurgencies* (The Scarecrow Press, Inc., Lanham, Maryland, 1996).
39. S.Ogura, *Document: the Whole History of the Vietnam War* (Iwanami Shoten, Tokyo, 2005) (in Japanese).

40. S.C.Tucker.(ed), *The Encyclopedia of the Vietnam War, Volume 3* (CABC-CLIO, LLC, Santa Barbara, California, 2011).
41. K.Shimizu, *A Chronological Table of the Vietnam War 1940-1975* (1983) (in Japanese).
42. S.Clissold (ed), *A Short History of Yugoslavia: from Early Times to 1966* (Cambridge University Press, 1966).
43. S.Clissold,.*A History of Yugoslavia: Enlarged Second Edition* translated by K.Tanaka.et al. into Japanese (Koubunsha, Tokyo, 1993).
44. M.Andou, *A Chronological Table of Present China 1941-2008* (Iwanami Shoten, Tokyo, 2010) (in Japanese).
45. D.-H.Tang, X.-P.Han, B.-H.Wang, Physica **A389**, 2637 (2010).
46. P.Bak, *How Nature Works* (Copernicus, NY, 1996).
47. G.F.Knoll, *Radiation Detection and Measurement, 4^{th} edition* (John Wiley & Sons, Inc., Hoboken, New Jersey, 2010).
48. N.Johnson, S.Carran, J.Botner, K.Fontaine, N.Laxague, P.Nuetzel, J.Turnley, B.Tivnan, Science **333**, 81 (2011).
49. T.Pollmann, Memory & Cognition **26**, 320 (1998).
50. H.Ebbinghaus, *Memory: a Contribution of Experimental Psychology* translated by H.A.Ruger and C.E.Bussenins (Teacher College, Columbia University, N.Y.,1913).