Physica A 392 (2013) 4127–4133

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Morphological similarity of road networks and cracks

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HIGHLIGHTS

- We study the possible similarity of road networks and cracks in their morphology.
- A desiccation model with double-layered cellular meshes is introduced.
- Simulation reveals the various patterns of real road networks and cracks.
- Simulation indicates the similar mechanism acting on their formation.

ARTICLE INFO

Article history:
Received 25 November 2012
Received in revised form 25 March 2013
Available online 30 April 2013

Keywords:
Road network
Crack pattern
Morphological similarity
Cellular mesh model
Simulation

ABSTRACT

An investigation was made regarding to what extent road network patterns are reproduced by a crack model from a viewpoint that they seem to resemble crack patterns in morphology. A desiccation model using double-layered cellular meshes was considered with the parameters representing the anisotropy of the material and the coarseness of grains, together with the introduction of singularities in points and in lines. The model can generally reproduce the real crack patterns and the road network patterns of cities with characteristic morphology by appropriately choosing the values of parameters, indicating that the similar mechanism acts on the formation of road networks and cracks of material although the relevant scales of space time differ from each other. Factors which make the road networks more complex and irregular in morphology were also investigated.

1. Introduction

How have the road networks through which we move everyday been formed? The formation mechanism of road networks such as assembling sets of roads, or the inevitable appearance of their morphological pattern has scarcely been considered even in a qualitative manner. In fundamental processes regarding the movement of mankind and the resultant formation of road, several factors have been considered as the outlook to a destination and the work required for getting to the destination [1–4]. When we take all these factors into account, however, it seems to be unable to obtain a general view or an appropriate model for the formation of the road network.

When we observe maps of the road network around big cities, we are seized with a feeling that the complicated web pattern of roads may be originated from some natural phenomenon in spite of the fact that it is a certain result of human activity. With regard to the road network within 200 km around Paris, for instance, an irregular pattern of network is formed as a superposition of trunk roads radially extended from the center of Paris on the outskirts roads which are almost perpendicular to the radial ones and concentric in many folds surrounding Paris. With approaching the center of Paris, such a road pattern becomes minute and increases its complexity. It reminds us of just the pattern of crack formed on a glass plate when a bullet is shot on it.

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0378-4371/$ – see front matter
http://dx.doi.org/10.1016/j.physa.2013.04.042

Although the road networks in medieval cities, or the shape and arrangement of the blocks of building and the interval between them appear at first glance to stochastically distribute and irregularly extend, they seem to have some sort of morphological regularity on close looking, which is common to all cities. In medieval cities such as Beaune, Toulouse, Albi, Milano, Bologna, Modena, Ferrara, etc., for instance, their central parts are generally held by a church or a castle standing on a slightly elevated hill, and small-scale building blocks irregularly extend outward from them or plazas near them as if they are the nuclei of the cities. Also in many Mediterranean cities in the Iberian Peninsula such as Sevilla, Cordoba, Valencia, Evola, etc., the church, the castle, the religious house or the palace, along with the castle wall, stand as an absolute and immovable point (or line) so that a plural number of roads radially extend from them and the road network is formed as a superposed structure of them on the plural number of roads almost perpendicularly crossing with them. These all indicate the church or the castle as a point where forces or stresses are accumulated on it, that is, a singular point, and the structural pattern of roads seems as if it is a crack pattern originating from such a singular point. On the other hand, in the area of Jewish residence in those Iberian Peninsula cities, complicated winding together with dead ends are also seen in the road structure. Such a morphology strongly reminds us of the crack appearing on a clay plate during the process of desiccation [5].

Moreover in the network structure of medina in old towns in Islamic cities such as Fes, Marrakech, Casablanca, etc., such complexity and irregularity can be said to reach an extremity. In those medinas, superposed structures of road are realized which consist of the plural number of main roads connecting public buildings such as the casbah, the mosque, the palace, the madrasa and the souq, the secondary roads almost perpendicularly branched from the main road, and the tertiary roads, often with dead ends, also perpendicularly branched from the secondary ones. Such an orthogonal arrangement of multiple constituents can also be seen in a Mediterranean city as Venezia which is unrelated to Islam, suggesting the participation of a certain hierarchical process during the formation of the road network. When we say the network structure as a resulting phenomenon of a hierarchical process, we are reminded of the mechanism of crack formation on clay plates and ceramics [6].

In the Islamic society cited above, residences are constructed by the guideline called Hadith [7,8]. According to this, a private residence is formed at the end of an already formed residential block on the micro and isolated spirit which regulates the individual relation only to the neighboring residences such as the local avoidance of malodor and noise, or the use of space above the road just in front of the residence. Namely a new road develops depending only on the microscopic condition at the top end of the road. Such a mechanism reminds us of Griffith's theorem of crack [9]. Moreover it well resembles the situation of some type of cellular automaton where a certain region stochastically develops under the subjection of some local condition finally to result in a macroscopic structure.

Thus, the road network as a resultant of human activity and the crack patterns in nature have a positive resemblance to each other, suggesting a possible interpretation by the similar mechanism. When we consider *social stress* as a sum of psychological stress originating from individuals in the society, such social stress increases as the root becomes to be a detour to their destination, whereas it decreases as a short cut is developed in such a manner as to minimize the effort to get to the destination. The realization of roads convenient for all people, therefore, means the reduction of social stress, and this corresponds just to the release of mechanical stress on a plane by forming cracks. The edifices such as the church, the school or the market draw the public so that the social stress accumulates around those functions. This makes them the social singularities from or to which roads are formed in a least-effort manner to or from the outer city area, just in the radial direction to release the social stress.

In what follows we therefore investigate the road network with a kinetic model of crack and study a problem what variation of the factors which make the crack can lead to the structure of the road network. We also discuss the probable fluctuations added on the ideal crack to lead to the real morphology of the road network.

2. Model

We adopt the cellular mesh model [10–13] as a simulation model for the crack formation and propagation. Considered here is a situation such that the cracks are formed on a water-containing square plane with sizes $(Lx, Ly) \equiv (L_0, L_0)$, which shrinks by the desiccation of water molecules. We then equally divide the plane into $n \times n$ meshes with a side length $\Delta L = L_0/n$. Every node of the mesh is assumed to be connected by bonds to the eight neighboring nodes as shown in Fig. 1. The node is a representative point of the material within the area of $(\Delta L)^2$, and the bond corresponds to a sort of elastic spring with a spring constant k. The material considered is of double layers with a surface and a base and with a thickness ΔL, and connected by the same bond as on the surface between the surface and the base. As the water molecules intermingled in the surface layer evaporate with time, the stress is gradually accumulated in the material. Such a situation may be represented by gradually increasing the spring constants of the surface bonds. The force acting on the surface node (i, j) at a time t is given by the sum of the stretching forces originating from the eight Moor bonds on the surface $\Sigma \vec{F}^t_{ij}$ and the similar stretching forces $\Sigma \vec{H}^t_{ij}$ by the bond connecting to the base plane. At a time $(t + \Delta t)$ after the evaporation of water molecules, the node (i, j) moves to a new position $\vec{X}(i, j)^{t+\Delta t}$ where those forces balance, that is

$$\vec{X}(i,j)^{t+\Delta t} = \vec{X}(i,j)^t + \frac{1}{4}\left(\sum \vec{F}^t_{ij} + \sum \vec{H}^t_{ij}\right) \cdot (\Delta t)^2 .$$

Here the mass of the node is assumed to be unity and a constant force is assumed to act during Δt with an average strength. The node on the base plane $\vec{X}_0\,(i, j)$ is also assumed to have a limited slippery characteristic to substrate with a relaxation

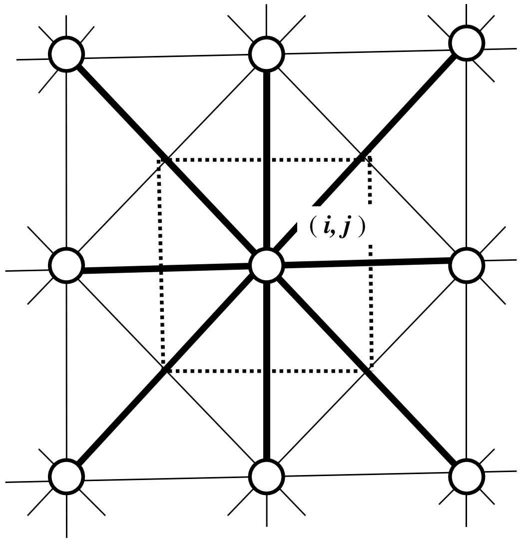

Fig. 1. Situation of nodes and bonds surrounding the node (i, j) which is a representative point of the square region given by dotted lines. Nodes and bonds are respectively shown by open circles and thick lines.

time T, that is

$$\vec{X}_0(i,j)^{t+\Delta t} = \vec{X}_0(i,j)^t + \frac{\Delta t}{4T}\left(\sum \vec{f}_{ij}^{\,t} + \sum \vec{h}_{ij}^{\,t}\right)\cdot(\Delta t)^2$$

where $\vec{f}_{ij}$ is the force on the base, and $\vec{h}_{ij}$ the force by the bond connecting to the surface plane. The difference of positions between $\vec{X}\,(i,j)$ and $\vec{X}_0\,(i,j)$ originates a stretch between the surface and the base which is reflected by the forces $\vec{H}_{ij}$ and $\vec{h}_{ij}$ so that $\vec{X}_0(i,j) = \vec{X}(i,j)$ and $\vec{H}_{ij} = \vec{h}_{ij} = 0$ in the case of $T = 0$.

We assume for simplicity that the water evaporates only from the surface, leading to negligible influence on the inner constituents. As a result of such evaporation, each bond length changes and the bond with a tension exceeding a critical strength or with a maximum tension is broken. Such a process occurs on all nodes and bonds at the same time and continuously in time. On the four boundary lines of the plane $(0, Ly)$, $(Lx, 0)$, (L_0, Ly) and (Lx, L_0), where $Lx, Ly \in [0, L_0]$, either condition is assumed for the material as (1) it satisfies a free boundary condition, or (2) it is bound by the lines so that it cannot move in the direction perpendicular to the lines. To take into account the effect of inhomogeneity of material originating from the mix of impurities and defects and the effect of the anisotropy of the material, the initial strengths of bonds are randomly determined on the assumption that the average spring constant in the direction of Neumann neighbors of a node is $\langle k\rangle_0$, whereas it is $g\langle k\rangle_0$ in the diagonal directions. Moreover both of these constants are assumed to have a normal distribution with a standard deviation s. Here the quantity g is a parameter representing the anisotropy of the material which has a value around $2^{-1/2}$.

The simulation procedure is as follows.

(1) Each bond strength on the surface is increased by a factor $(1 + d)$ at a discrete time t. This owes to the assumption that the water molecules between lattices homogeneously evaporate in space and in time from the surface.
(2) The force acting on each node on both sides of the surface and base is derived.
(3) An equilibrium position is derived anew for each node.
(4) The new length of each bond, and hence the stress on each bond is derived.
(5) One bond on the surface with the maximum stress is cut.
(6) By proceeding the time by Δt, an iteration is made from the process (1).

Hereafter we set $L_0 = 1$, $\langle k\rangle_0 = 1.0$, $k(0, 2.0)$, and $\Delta t = 0.05$. The parameters are d, g, s, T, and the mesh number n. In the region considered, we usually assume the existence of singularities in point or in line. The singular line corresponds to the boundary surrounding the region, and immovable singular point(s) are considered in the case of no boundaries. Those singular points correspond to pebbles immersed in the material, for instance.

3. Simulation results

Fig. 2 shows the propagation of cracks with time under the condition of $T = 10$. Cracks in Fig. 2.1(a)–(c) grow from boundaries toward the center. Since we set $s = 0.5$ in this case, it corresponds to a considerably high randomness in direction for a crack to propagate (namely, the randomness in direction for the highest stress field) at its growing tip. In reality this

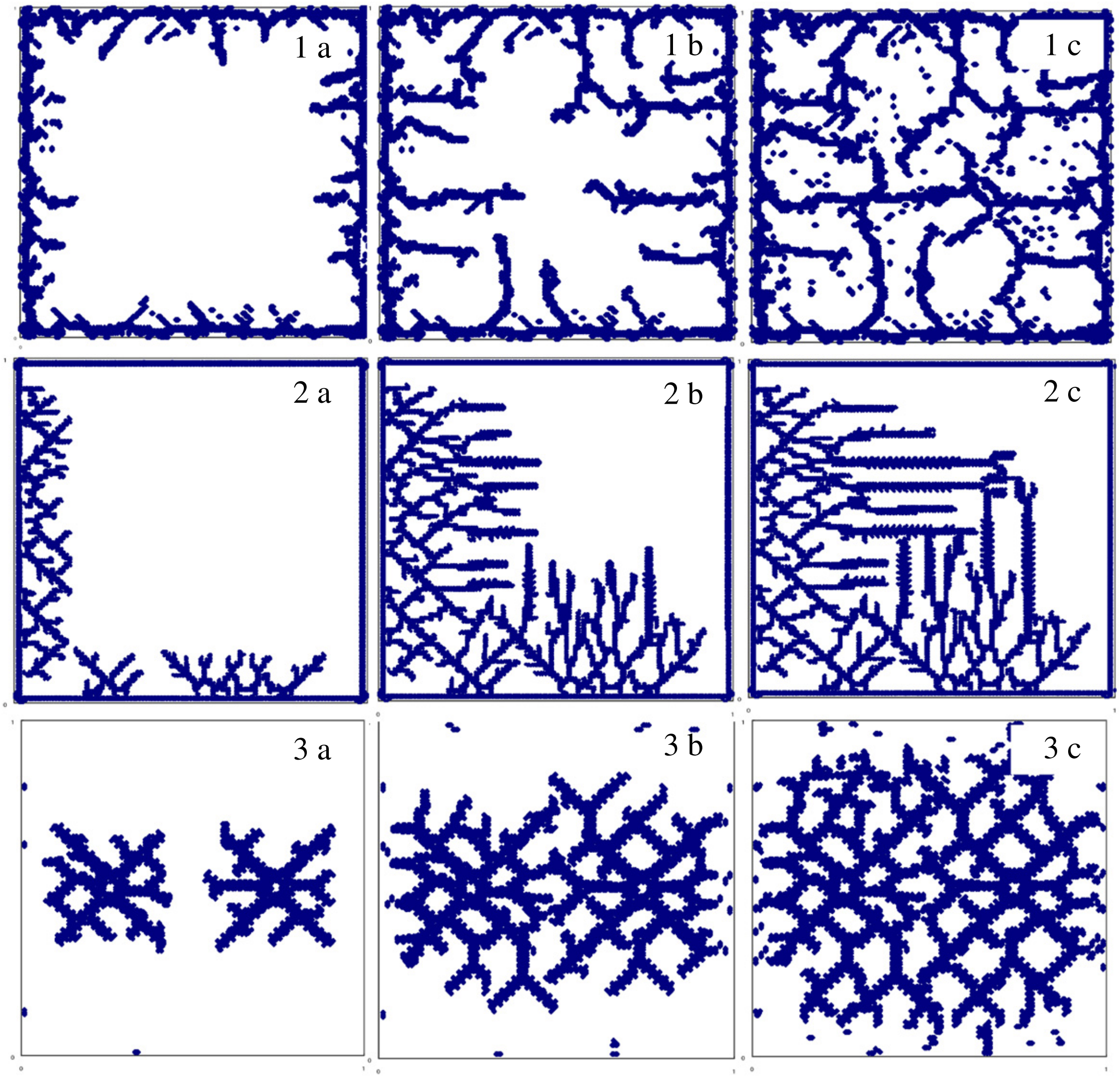

Fig. 2. Propagation of cracks with time. 1(a) At $t = 2000$ with $d = 0.001$, $g = 2^{-1/2}$, $s = 0.5$, $n = 100$ and with boundaries, where time t is in a unit of Δt; 1(b) $t = 3000$; 1(c) $t = 4000$; 2(a) at $t = 3000$ with $d = 0.0005$, $g = 0.723$, $s = 0.01$, $n = 120$ and with boundaries; 2(b) $t = 5200$; and 2(c) $t = 6000$; 3(a) at $t = 700$ with $d = 0.001$, $g = 0.73$, $s = 0.01$, $n = 60$ and with two singular points in the inner region; 3(b) $t = 1400$; 3(c) $t = 2000$.

case just corresponds to the material such as a clay plate with coarse grains, and our results seem well to reproduce the crack features of such a type [14].

Fig. 2.2(a)–(c) are of the same boundary condition as in 2.1(a)–(c), but with finer grains and with a slower rate of desiccation such as $d = 0.0005$. Cracks like a net grow from two sides of boundaries toward the central region. Such a pattern reminds us of the growth of crystal dendrite [15,16]. With the propagation of cracks, their forms change to the pattern of swords to extend toward the opposite sides of boundaries.

In Fig. 2.3(a)–(c), there exist two singular points on a horizontal center line with a distance $L_0/2 = 0.5$ between them. Cracks begin to grow from these singularities. Since $s = 0.01$ in this case, the pattern corresponds to the cracks on a clay plate with fine grains. Observed cracks of this type are seen in Fig. 3(a).

We next investigate the possible reproduction of the real network features of roads when we vary the values of those parameters. Fig. 4 shows two examples with $g = 2^{-1/2}$, $d = 0.001$, $s = 0.01$, and $T = 500$, but with different boundary conditions. Fig. 4.1(a) assumes two singular points on the center of boundaries on both sides. Primary cracks propagate from these singularities to combine together at around the center, leading to the secondary and tertiary cracks perpendicularly growing to the respective primary and secondary cracks finally to result in a quite irregular pattern. As a city with such a pattern, we think of Marrakech as shown in Fig. 4.2(a), for instance. Fig. 3(b) also shows an example of the ceramic cracks of this type.

Fig. 4.1(b) shows a pattern under the condition of surrounding boundaries, namely the singularity in line from which primary cracks grow almost perpendicularly. The secondary cracks grow in the inner region from those primary ones.

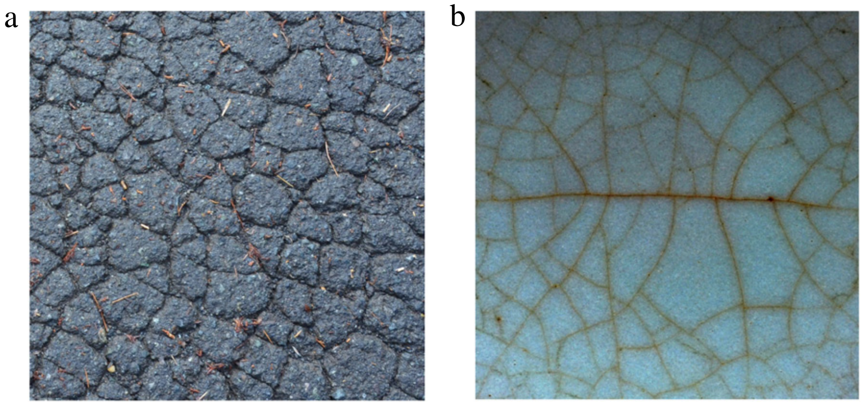

Fig. 3. Observed patterns of cracks on (a) an asphalt road, and (b) ceramics.

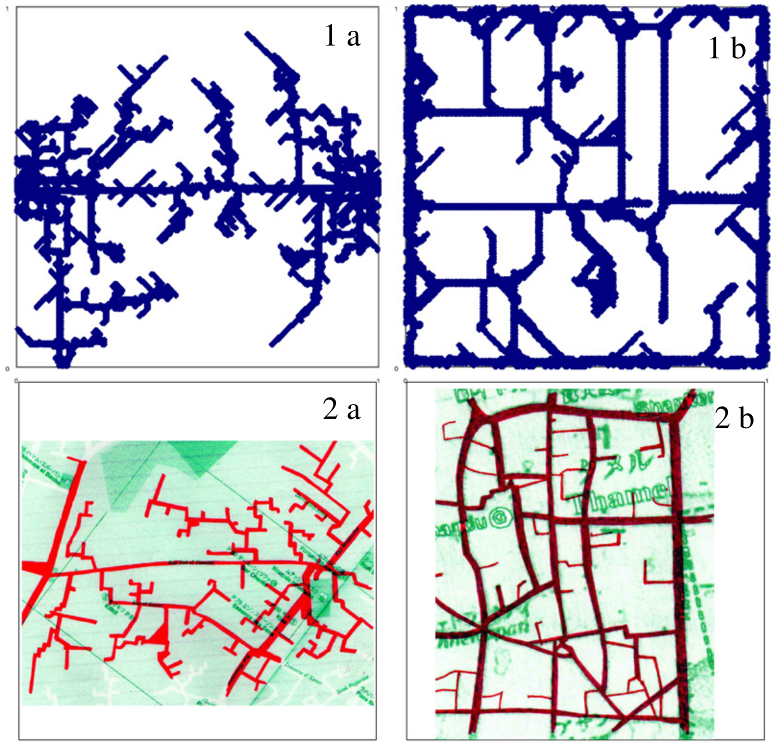

Fig. 4. Examples of patterns realized with our crack model and patterns of real road networks depicted by red lines. 1(a) At $t = 4500$ with $g = 2^{-1/2}$, $T = 500$, $n = 150$ and with a singularity on each side of boundaries; 1(b) at $t = 5400$ with $g = 2^{-1/2}$, $T = 500$, $n = 120$ and with the boundaries; 2(a) road network in old Marrakech; and 2(b) road network in present Kathmandu. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The difference in condition from the previous Fig. 4.1(a) is the form of singularity. We may think of Kathmandu shown in Fig. 4.2(b) as a city of this type.

Fig. 5 represents examples under the condition of one or two singularities with $T = 0$. Fig. 5.1(a) is for the case of two singular points set on a horizontal center line with a distance 1/3. The road network in Sevilla [17] shown in Fig. 5.2(a) is an example of the realized pattern. When we set only one singularity at the center, on the other hand, we obtain Fig. 5.1(b) where the values of g and n also differ from the previous Fig. 5.1(a). A little fine structure appears around the singular point with cracks radially extending from it, whose pattern reminds us of the roads in Paris [18] shown in Fig. 5.2(b). With the same parameters except for n which makes the mesh coarser than the previous one 5.1(b), we obtain Fig. 5.1(c). To make the mesh coarse corresponds to the approach to an object. As the approach to the singularity (1(a) $\rightarrow$ 1(b) $\rightarrow$ 1(c)), therefore, we can see the finer and finer pattern of crack around it. An example of the road network as such can be seen in the medieval Noerdlingen [19] shown in Fig. 5.2(c).

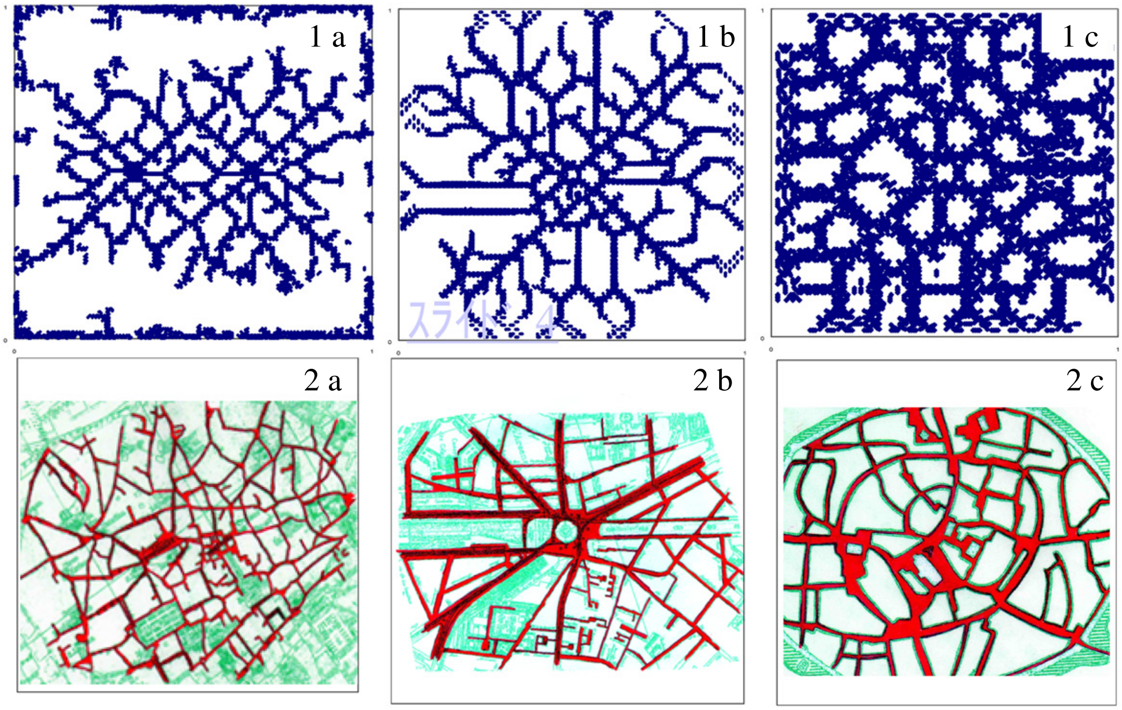

Fig. 5. The same as Fig. 4, but, 1(a) at $t = 4000$ with $g = 0.73, T = 0, n = 100$ and with two singular points; 1(b) at $t = 3000$ with $g = 0.723, T = 0, n = 80$ and with one singular point; 1(c) at $t = 1600$ with $g = 0.723, T = 0, n = 40$ and with one singular point; 2(a) road network in Sevilla in 1899; 2(b) road network in Paris in 1909; and 2(c) road network in medieval Noerdlingen.

The major parameters which determine the crack pattern or the road network pattern in Figs. 4 and 5 are the type and the number of singularities, g, T and n. Singular points and lines in urban systems correspond to the public functions such as the public buildings, the plaza, the market or the castle wall which are all fixed in time. We can understand that the existence of such functions is one of the major factors to determine the pattern of the road network. The increase of the quantity g enhances the probability for the road to propagate in the oblique directions. A large relaxation time T indicates that the base of the clay plane is bound more strongly to the underlying material so that the crack width becomes narrow with increasing T, thus suggesting the width of the road to become narrow with T. Since the quantity n, on the other hand, is a parameter giving the extent of region considered around the singularity, a small n corresponds to looking at the region closer to the singularity and hence a fine structure of the road network near the urban singularity.

4. Discussion

Patterns shown in Figs. 4 and 5 seem to be quite similar to the real road networks in some cities even if they are not precise reproduction. This indicates the fundamental resemblance in the relevant mechanism, and the possible explanation for the origin of the road network by a crack model to a certain extent. Notwithstanding this, the complexity and irregularity in the real road networks seem to have rather different appearance from the crack patterns in nature. This may be due to the intervention of some characteristic factors in the case of road network such as geographical, socio-historical and unconscious human factors.

Geographical factors which affect the formation of the road network are the obstacles such as the mountain, the river and the coast. We can easily imagine that such obstacles block the free expansion of road to result in a different feature in morphology from the nature. Moreover when the road is formed on an uneven field with rise and fall such as the mountain, the hill and the valley, it may be in the direction along contour lines or almost perpendicular to those lines connecting to each other in a form of short cut, or along the root of water flowing down from the heights, that is the stream line. The medieval castle city Toledo, for instance, was constructed on a rocky mountain, whose road network pattern strongly indicates to have been subjected by the topography of the mountain.

The second cause to give the complexity and irregularity in road networks is the socio-historical factor. The medieval fort city was generally formed in an enclosed castle wall, and in its inner space a complicated network of road was constructed against the invasion of foreign enemy. The casbah in Islamic cities is an example of this. Moreover when different races or tribes with different religions such as the Muslim and the Christian gathered together to form a living space for their own race or tribe, they usually constructed a complex and irregular network of road to defend against the attack of other races or tribes. Alfama in Lisboa and Albaicin in Granada which were the residential areas of the medieval Moors are the examples of this.

The third cause for the road network to increase its irregularity is the human factor. In the process of a primitive path or a short cut to gradually become a formal road [1–3], it is important to consider the stochastic and non-linear human factors such as the psychological desiring of a shorter way, the psychological height of barrier existing along the way of short cut, the perception of the trail which was already formed and survived at that time, and the visibility of a destination. Although such factors do not play any roles in natural cracks, they unconsciously and undoubtedly appear in the formation process of road to lead to the complexity in the road network.

Thus even in the case where the formation mechanisms are similar in different phenomena in trans-culture fields, their appearances may not be necessarily similar due to the ''contamination'' by noises or perturbations characteristic to one or both of the phenomena. Two or more than two phenomena similar to each other in their morphology, however, are probably of high similarity also in fundamental mechanism of their formation. There exist many examples suggesting the working of a similar mechanism among phenomena in seemingly different scales in space time and in discipline, from a scale of microorganism to a scale of the earth, such as the network of slime mold to pursue nutriment and the optimum networks of railroad [20,21], the veins of a plant leaf and the web pattern of rivers [15,22], the trail by ants and the pattern of discharge in the air [15,23], the columnar structure of starch and that of basalt [24,25], etc. This makes us reconfirm that the law of nature universally extends, transcending the space time like the ether. On the practical side, we can take notice of the possibility that the new appearance of one phenomenon may be forecasted from the feature of another phenomenon by using one as a simulator of the other.

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